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III.

30. PERFECT GAS FLOWING FROM ONE VESSEL TO ANOTHER.—Let p_1 v_1 t_1 q_1 be the pressure, specific volume, temperature and total weight respectively of the gas in the first vessel in its initial state, and let p_2 v_2 t_2 q_2 designate the initial state in the second vessel.

Let same gas flow from the first vessel to the second, and let the final state of the first vessel be designated by the subscript 4. and the final state of the second vessel by 3.

$$\therefore\ q_1-q_4=q_3-q_2 \quad . \quad . \quad (156)$$

is the quantity of gas which has flowed from the first vessel to the second,

also, $$q_1v_1=q_4v_4 \quad . \quad . \quad . \quad (157)$$

and, $$q_3v_3=q_2v_2 \quad . \quad . \quad . \quad (158)$$

are the volumes of the first and second vessels respectively.

Also, if no heat is imparted to or withdrawn from the gas, the expansion in the first vessel is adiabatic, but the second vessel has the energy imparted to it which is withdrawn from the first vessel.

$\therefore$ by (129) or (135), and by (157)

$$\frac{p}{p_1}=\left(\frac{v}{v_1}\right)^{-n}=\left(\frac{q}{q_1}\right)^{n}=\left(\frac{t}{t_1}\right)^{\frac{n}{n-1}} \quad . \quad . \quad (159)$$

express the state in the first vessel at any time.

By (10), $$\int_1^4 qdw=\int_1^4 qpdv \quad . \quad . \quad (160)$$

is the work of expansion in the first vessel from the state 1 to the state 4 performed in driving gas into the second vessel. Since the expansion is adiabatic, this work can be expressed by help of (159) in terms the single variable p.

$$\therefore\ \int_1^4 qdw=-\frac{q_1v_1}{n}\int_1^4 dp=\frac{q_1v_1}{n}(p_1-p_4) \quad . \quad . \quad (161)$$

is the work of expansion gained by the second vessel. It can be expressed in terms of either other variable by help of (159).

Again, $$\int_1^4 sdq=k_v\int_1^4 tdq \quad . \quad . \quad (162)$$

is the energy contained in the gas gained by the second vessel in the form of heat. But by (121), (109) and (159)

$$\int_1^4 sdq=-\frac{q_1v_1}{n(n-1)}\int_1^4 dp=\frac{q_1v_1}{n(n-1)}(p_1-p_4) \quad . \quad . \quad . \quad (163)$$

The total energy transferred from the first to the second vessel is the sum of (161) and (163)

* Figs. 2, 3 and 4 which should have been inserted in the April number of the Magazine will be found at the end of this article.

$$\therefore \int_1^4 q\,dw + \int_1^4 s\,dq = \frac{q_1 v_1}{n-1}(p_1 - p_4) \quad . \quad (164)$$

This can be otherwise readily shown, for

$$\text{by (141)} \qquad n k_v \int_1^4 t\,dq \quad . \quad . \quad . \quad (165)$$

is the total energy transferred. But (165) is n times the last member of (162), hence its value is n times the last member of (163), which was to be proved.

By writing the subscript 4 in (159) we find the relations between the quantities p_4, v_4, t_4, q_4, which designate the second state of the first vessel.

To find the second state of the other vessel; the energy of sensible heat in the initial state of the gas in this vessel, is by art. 23,

$$k_v\, q_2 t_2 = \frac{q_2 v_2 p_2}{n-1} \quad . \quad . \quad (166)$$

and the energy imparted by the first vessel is (164): the sum (164) and (166) must be equal to the total energy of sensible heat of the gas in the state 3, which is $k_v\, q_3\, t_3$. Now divide by k_v and we have

$$q_3\, t_3 = q_2 t_2 + q_1\, t_1 \left(1 - \frac{p_4}{p_1}\right) \quad . \quad (167)$$

in which q_3 is given by (156), hence t_3 can be found from (167)

Also, by (9), $$\frac{p_2 v_2}{t_2} = \frac{p_3 v_3}{t_3}$$

$$\therefore \text{ by (158)}, \qquad \frac{p_2}{q_2 t_2} = \frac{p_3}{q_3 t_3} \quad . \quad . \quad (168)$$

Now divide (167) by $q_3\, t_3$

$$\therefore \text{ by (167)}, \frac{p_3}{p_2} = 1 + \frac{q_1 t_1}{q_2 t_2}\left(1 - \frac{p_4}{p_1}\right) \quad . \quad (169)$$

from which p_3 can be found. We have now shown how to find each of the variables p_3, v_3, t_3, q_3 in terms of known quantities, when a given quantity $q_1 - q_4$ flows from one vessel to the other.

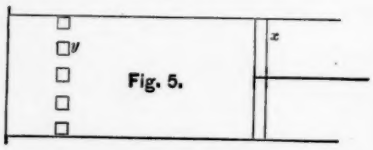


Fig. 5.

31. Hot Air Engines.—In Fig. 5 let x be a solid piston working in a cylinder by which the volume of the air, or other perfect gas, contained in the cylinder and in vessels connected with it may be made to vary. Let x be called the working piston. And let y be a perforated piston also working in a cylinder. On moving y air is transferred through it from one side of it to the other. Let y offer no resistance to the passage of air through it, so that the pressure of the entire mass of air confined by the working piston x is the same on whichever side of y it may be. The piston y serves to divide the whole confined air into two parts, which we shall suppose to be at different temperatures but at the one pressure. Let the whole space at the left of y be called the furnace, and let the air in the furnace be all at the one temperature. Call the space at the right of y the clearance, and let all the air in it be at another temperature. The piston y is the supply piston of the furnace, since by it air is transferred from the clearance to the furnace.

It is seen that by moving x the space occupied by the clearance is altered, but the volume and pressure are varied in both furnace and clearance. By moving y the total volume of furnace and clearance is unchanged, but the space occupied by one is increased at the expense of the other, while the air transferred from one to the other has undergone a change of temperature, and consequently there has been also a change of the specific volumes and pressures in both furnace and clearance.

The simple arrangement just described will enable us to discuss various different forms of air engines, without regard to the special contrivances which may be employed for imparting and rejecting heat, or for causing the working air to pass through either of the possible cycles which will be assumed.

We shall confine our attention principally to the efficiency of the various cycles assumed for the working substance, as the determination of the volume swept through by the working piston and other practical matters of the first importance in actually designing an engine must be omitted in the present brief, theoretical discussion.

One special contrivance, called the regenerator, should, however, be noticed in this connection, on account of the important role it plays in modifying the cycle of changes through which the

air passes. When an air engine is supplied with a regenerator we shall conceive that the piston y is this regenerator, which in that case will be supposed to be constructed in such a manner as to store up all, or a part of the heat of the air passing through it from the furnace to the clearance, and to restore it to the air passing from the clearance to the furnace. The regenerator may be conceived to consist of many sheets of wire netting. If no conduction of heat occurs in the regenerator, the heat will be stored and restored at the same temperature, so that the passage of the air through it may be regarded as being attended by an adiabatic variation of the working substance, when by working substance is meant both the air and the material of the regenerator.

32. Working Air and Cushion Air.—That part of the air in any air engine to which heat is imparted, and from which heat is rejected is called the working air. That part of the air which experiences only adiabatic changes is called the cushion air, and being alternately expanded and compressed, performs as much work during expansion as is expended in compressing it. Hence the external work performed depends in no way upon the cushion air. In practice, part of the cushion air remains in the furnace, but most of it is usually in the clearance. The cushion air however has an important effect on the size of the cylinder. Since it expands at the same time as the working air, the proportion of cushion air to each unit of working air should be as small as possible, in order that the space swept through by the working piston x may be as small as possible.

33. Air Engine with Adiabatic Expansion.—1°. Let both x and y move to the right in such a manner that the initial temperatures t_1 and t_2 of the clearance and the furnace are unchanged during this first operation: hence $p_1=p_2$ is also unchanged. This operation involves heating all the air which passes through y into the furnace from t_1 to t_2 at the instant it passes through y.

The total amount of heat imparted to a unit of air in carrying it from the clearance to the furnace at constant pressure is

$$\text{by (115)}, \qquad h_2-h_1=k_p\,(t_2-t_1) \quad . \quad (170)$$

2°. Let x and y move to the right in such a manner that an adiabatic expansion occurs in both the furnace and clearance without transfer of air through y.

Since the pressure, the specific volumes and the temperatures t_1 and t_2 were unchanged during the first operation, we have,

$$p_1=p_4, \text{ and } p_2=p_3 \quad \therefore \quad (171)$$

and also, by (171) and (135),

$$\frac{t_4}{t_1}=\left(\frac{v_4}{v_1}\right)^{1-n}=\left(\frac{p_4}{p_1}\right)^{\frac{n-1}{n}}=\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}=\left(\frac{v_3}{v_2}\right)^{1-n}=\frac{t_3}{t_2} \quad . \quad . \quad . \quad (172)$$

in which the subscript 4 designates the state in the clearance at the end of the second operation, and 3 the state in the furnace.

3°. Let x and y move toward the left in such a manner that the temperatures t_4 and t_3 remain constant during this third operation. This involves cooling all the air from t_3 to t_4 at the instant it passes through y into the clearance.

The total amount of heat rejected in transferring a unit of air from the furnace to the clearance is

$$\text{by (115)}, \qquad h_3-h_4=k_p\,(t_3-t_4) \quad . \quad (173)$$

4°. Let x and y move to the left in such a manner that an adiabatic compression occurs without transfer of air through y, and let the compression continue until at the end of this fourth operation the air is restored to its initial state in both furnace and clearance. Since the pressure, the specific volumes and the temperatures t_3 and t_4 were not changed during the third operation, (172) applies to this fourth operation when the subscripts 4 and 1 designate the states of the clearance at its beginning and end respectively, while 3 and 2 designate the corresponding states of the furnace.

The total external work performed must be the heat imparted (170), minus the rejected (173).

The efficiency ε, is the ratio of the work performed to the heat imparted.

$$\therefore \quad \varepsilon=\frac{(t_2-t_1)-(t_3-t_4)}{t_2-t_1}$$

$$\therefore \text{ by (172)}, \quad \varepsilon=\frac{t_2-t_3}{t_2}=\frac{t_1-t_4}{t_1} \quad . \quad (174)$$

Hence by art. 18, an air engine working in the cycle, above indicated, is equiva-

lent in efficiency either to a perfect engine between the temperatures t_3 and t_2 of the beginning and end of the expansion of the working air, or to a perfect engine between the beginning and end of the expansion of the cushion air. As t_3 is the temperature of the source and t_4 that of the refrigerator, the efficiency above given would in all cases be less, and practically much less than the efficiency of a perfect engine between these limiting temperatures, which efficiency is by art. 18,

$$\varepsilon = \frac{t_3 - t_4}{t_3} \quad . \quad . \quad . \quad . \quad (175)$$

The only manner in which the value of (174) can be made to approach (175) is by making t_1 approach t_3, and hence t_4 also approach t_3, but this would require a very large working cylinder in proportion to the power developed, a difficulty which has been inherent hitherto in the designs of air engines.

Now let a regenerator be applied to an air engine working in the cycle just described, in such a manner that it stores up during the third operation (during which air is carried from the furnace at a temperature t_3) a sufficient quantity of heat so that during the first operation the regenerator shall restore the air as it enters the furnace to the same temperature t_3.

$\therefore$ by (115), $\quad k_p\,(t_3 - t_1) \quad . \quad . \quad . \quad (176)$

is the heat stored and restored by the regenerator per unit of working air.

Also, $\quad k_p\,(t_2 - t_3) \quad . \quad . \quad . \quad (177)$

is the heat imparted by the furnace,

and $\quad k_p\,[t_3 - t_4 - (t_2 - t_1)] \quad . \quad . \quad (178)$

is the heat rejected per unit of working air.

$$\therefore \ \varepsilon = \frac{(t_3 - t_2) - (t_1 - t_4)}{t_3 - t_2}$$

$$\therefore \text{ by (172)}, \quad \varepsilon = \frac{t_2 - t_1}{t_2} = \frac{t_3 - t_4}{t_3} \quad . \quad (179)$$

It is seen by comparing (179) with (174), that an air engine working in this cycle has its efficiency increased by a regenerator in case t_3 is greater than t_1, but its efficiency is diminished in case t_3 is less than t_1. Since, however, as has been previously stated, t_1 must be almost equal to t_2 in case the efficiency approaches the case of perfect efficiency contemplated in (175), it appears that in general a regenerator would be useless in an engine having a cycle of this kind, for t_3 should be less than t_1.

The cycle of this engine is imperfect because the temperature of the working substance is increased by imparting heat to it, and lowered by losing heat, whereas for a perfect cycle, by art. 18, the heat should be imparted along one isothermal and rejected along another, which is not the case in the cycle which has been discussed.

34. Air Engine with Isothermal Expansion.—Let the cycle of operations which the working air is made to undergo be such that the temperature t_2 of the furnace is constant during all the operations, it being that of the source of heat; and the temperature t_1 of the clearance also remains constant during all the operations, it being that of the refrigerator. The first and third operations will then be the same as in art. 33, but in the second and fourth operations heat is imparted and rejected in such a manner that the working air undergoes isothermal expansion and compression instead of the adiabatic variations supposed in art. 33.

By (115), $\quad k_p\,(t_2 - t_1) \quad . \quad . \quad . \quad (180)$

is the heat imparted during the first operation. The heat imparted to a unit of the working air during the isothermal expansion is

$$\text{by (119)}, \ t_2(e_1 - e_2) = c t_2 \log_e \frac{p_2}{p_1} \quad . \quad (181)$$

By help of (122) and (135) we have

$$t_2(e_1 - e_2) = k_p\, t_2 \log_e r \quad . \quad . \quad (182)$$

which is the heat imparted during the isothermal expansion from the initial pressure p_1 to the final pressure p_2.

$$\therefore \ k_p\,[t_2 - t_1 + t_2 \log_e r] \quad . \quad . \quad (183)$$

is the total heat imparted to a unit of the working air. Similarly

$$\therefore \ k_p\,[t_2 - t_1 + t_1 \log_e r) \quad . \quad . \quad (184)$$

is the total heat rejected per unit of working air during the third and fourth operations.

$$\therefore \ \varepsilon = \frac{t_2 - t_1}{t_2 + \dfrac{t_2 - t_1}{\log_e r}} \quad . \quad . \quad (185)$$

This cycle is more perfect as the ratio of expansion is increased.

By (135) the ratio of expansion $v_2 \div v_1$ is related to the quantity r in the following manner.

$$v_2 \div v_1 = r^{\left(\frac{1}{1-n}\right)} \quad . \quad . \quad . \quad . \quad (186)$$

Let a regenerator be applied to an air engine working in the foregoing cycle. Then the quantity of heat stored during the third operation and restored during the first operation per unit of air is expressed by (180), hence the total heat imparted by the furnace is during the second operation, and is expressed by (182).

Similarly, the total heat rejected is during the fourth operation and is got by changing t_2 into t_1 in (182). Let the temperature of the furnace and clearance be t and t' respectively.

$$\therefore \varepsilon = \frac{t-t'}{t} \quad . \quad . \quad . \quad (187)$$

is the efficiency of an air engine with a regenerator working in this cycle. As is seen, this engine is of perfect efficiency, and it is made so by the action of the regenerator, which fact confirms the statement made in art. 28 to the effect that the changes undergone in passing the regenerator are adiabatic, since no engine can have a perfect efficiency, as this one has, by art. 18, except its cycle consists of adiabatics and isothermals alone.

35. Ericsson's Hot Air Engines.—Two kinds of hot air engines have been put in practical operation by Ericsson. The first was constructed with a regenerator and the second without a regenerator.

It is difficult to know with certainty what cycle is described by the working air in these engines.

It has been usually assumed that Ericsson's engine with the regenerator had approximately an isothermal expansion, but that this was the fact appears more than doubtful, for it is known that air is a bad conductor of heat, and hence it would be very difficult, not to say impossible, with the arrangement of furnace which was actually employed to supply to the air during its expansion the heat necessary to prevent a fall of temperature. It seems probable that the expansion was more nearly adiabatic, but that during the outflow the air was again heated to its initial temperature, *i. e.*, to the temperature of the commencement of the expansion. The effect of this would be to cause the efficiency of the engine to have a value near that expressed by (179), instead of the perfect efficiency expressed by (187). This great loss of efficiency might probably be largely overcome by an arrangement of furnace such as has been proposed by Rankine * and employed in the hot air machine of Lemoine. †

A complete discussion, however, of this matter can only be made when we know the rate at which heat is imparted by the furnace at different parts of the stroke. If the working air is maintained at a constant temperature it is evident that heat flows through the walls of the furnace uniformly, for that flow is determined by a constant difference of temperature between the outside and inside of the walls through which the heat passes. An isothermal expansion can therefore be effected only when the parts of the stroke are so arranged as to require a uniform expenditure of heat. It is needless to say that such an adjustment is not that of a crank and piston stroke.

In Ericsson's engine with a regenerator, the compressed air was supplied to the furnace by a separate pump in such a manner that the air entering the regenerator from the pump was at nearly the same temperature as that issuing from the regenerator after giving up its heat to the regenerator. Under such circumstances the occurrence of any conduction or radiation of heat in the regenerator in the brief time during which it was necessary for it to retain the stored up heat, would result in a loss of efficiency.

The reason of this is that conduction or radiation is a process of transferring heat from a higher to a lower temperature without doing the work which by Carnot's principle it might be made to perform during such transference. It seems probable that the heating previously mentioned, which occurred during the outflow, increased the efficiency of the engine, and that a still further heating would be economical as tending to make up for the loss occasioned by

* Steam Engine. Art. 275.

† Novelle Mechanique Industrielle. L. Pochet, Paris, 1874, p. 176.

conduction and radiation in the regenerator.

In Ericsson's air engine without a regenerator, the very complicated mechanical arrangements employed in causing the stroke of the compressing pump do not permit any simple investigation of the rate at which heat is supplied, but evidently the cycle of the working substance lies between that of an adiabatic and an isothermal expansion, and the efficiency which is not large, has a value between that given by (174) and (185).

Ericsson's engines both draw their supply of air from the atmosphere and discharge it again into the atmosphere. The principal advantage of this arrangement is that by it all other provision for rejecting heat is rendered unnecessary, unless possibly it may be best to cool the condensing pump, whereas in case the same working air is used again and again some form of refrigerator must be employed. This usually involves the expenditure of work in pumping a considerable quantity of water.

The principal disadvantage of discharging air into the atmosphere is the small working pressure during the stroke which involves a very large working cylinder in proportion to the power developed.

36. Stirling's Hot Air Engine.—In the hot air engine constructed by Stirling the working air is introduced into the furnace with little or no change of volume, hence the pressure and temperature rise during its introduction, and in like manner fall during its discharge from the furnace.

1°. To find the efficiency of an engine in which the working air in Fig. 5 is transferred with increase of temperature from the clearance to the furnace at constant volume, and then expanded along an adiabatic, after which it is transferred with decrease in temperature from the furnace to the clearance, and, lastly, it is compressed along an adiabatic to its initial condition.

Disregard the work stored and restored by the cushion air, which will have no effect upon the final result, and let the subscripts 1, 2, 3, 4, designate the successive states above described, then

by (59), $h_2-h_1=k_v\,(t_2-t_1)$ . . (188)

is the heat imparted to a unit of weight of air in raising its temperature from t_1 to t_2, and $h_3-h_4=k_v\,(t_3-t_4)$ . . (189) is the heat rejected in cooling a unit of air from t_3 to t_4.

But since the second and fourth operations are adiabatic, and also

$$v_1=v_2, \quad \text{and} \quad v_3=v_4, \quad \text{by (129)}$$

$$\frac{t_2}{t_3}=\left(\frac{p_2}{p_3}\right)^{\frac{n-1}{n}}=\left(\frac{v_3}{v_2}\right)^{1-n}=\left(\frac{v_1}{v_4}\right)^{1-n}=\left(\frac{p_1}{p_4}\right)^{\frac{n-1}{n}}=\frac{t_1}{t_4} \quad . \; . \; . \quad (190)$$

$$\therefore\ \varepsilon=\frac{(t_2-t_1)-(t_3-t_4)}{t_2-t_1}$$

$$\therefore \text{ by (190)}, \quad \varepsilon=\frac{t_1-t_4}{t_1}=\frac{t_2-t_3}{t_2} \quad . \quad (191)$$

from which it appears that the efficiency of this cycle is the same as that found in (174), viz: that of a perfect engine working between the temperatures of the beginning and end of the expansion.

If a regenerator be applied to an engine working in this cycle the efficiency will be found, just as in (179), to be

$$\varepsilon=\frac{t_2-t_1}{t_2}=\frac{t_3-t_4}{t_3} \quad . \; . \; . \quad (192)$$

By comparing the cycle discussed in art. 33 with this, it appears that this has the advantage of performing the same work with a smaller working cylinder than that.

2°. Let the working air be received into and discharged from the furnace at a pair of constant volumes, as just considered, but let the expansion and the compression be each isothermal instead of adiabatic. Let the temperature of the source be t_2 and that of the regenerator t_1

then, $\quad k_v\,(t_2-t_1)+k_p\,t_2\log_e r \quad . \quad (193)$

and, $\quad k_v\,(t_2-t_1)+k_p\,t_1\log_e r \quad . \quad (194)$

are respectively the total heat imparted and the total heat rejected.

$$\therefore\ \varepsilon=\frac{t_2-t_1}{t_2+\dfrac{t_2-t_1}{n\log_e r}} \quad . \; . \quad (195)$$

is the efficiency of this circle, which is somewhat greater than that of the isothermal expansion whose value is expressed by (185). It likewise has the advantage of a smaller working cylinder.

If a regenerator be applied to this

engine, the heat stored in the third operation is restored during the first operation, and the cycle becomes one of perfect efficiency between the temperatures t_2 and t_1.

The same difficulties are in the way of our knowing the precise cycle of Stirling's engine as of Ericsson's. The working air has an expansion intermediate between an isothermal and an adiabatic. Which engine is theoretically more advantageous in form, Stirling's or Ericsson's, in case each uses the same air continuously is largely a matter of the arrangement of the furnace for heating the air, with the advantage apparently a little in favor of Stirling's engine. As has been before remarked, in such a system, the confined air should by at a high pressure, even at its greatest expansion, in order that the cylinder may be small. Furthermore, Stirling's engine cannot (as Ericsson's can) make the atmosphere its refrigerator. The same considerations apply to the conduction and radiation in the regenerator of Stirling's engine which have been previously adduced in regard to that of Ericsson's engine.

An interesting and valuable discussion of these and other air engines may be found in six articles published in *Engineering*, Vol. XIX (January to July, 1875), in which it is shown that the practical difficulties of construction were more of them overcome in Stirling's engine than in any other; the principal remaining difficulty having been the great heat to which the furnace was exposed which soon destroyed it. Yet if this can be remedied without its being necessary to decrease the temperature of the furnace, the ultimate superiority of air engines over steam engines, will be thereby rendered almost certain, since any such high temperature is not to be thought of in a steam engine by reason of the great pressure which necessarily accompanies it, but in an air engine the pressure at any temperature is a matter entirely within control. Another difficulty inherent in all air engines is the rise of temperature which occurs when the engine is temporarily stopped without stopping the fire.

37. Transmission of Power by Compressed Air.—If air be drawn from the atmosphere at an initial state expressed by $p_1\ v_1\ t_1$ and be driven by a compressing pump into a receiver in which its state is $p_2\ v_2\ t_2$, it is at first compressed from p_1 to p_2 and then driven at pressure p_2 into the receiver. The work performed during both these operations will depend upon the kind of compression adiabatic, isothermal or otherwise, through which the air passes, during the first operation.

The expression for the total work performed in forcing a unit of air into the receiver is

$$w = p_2 v_2 - p_1 v_1 - \int_1^2 p\,dv \quad . \quad . \quad (196)$$

in which the integral of $-pdv$ is the work of compression, during the first operation, $p_2\ v_2$ is the work during the second operation, and $p_1\ v_1$ is the assistance experienced by reason of the pressure of the external atmosphere.

1°. If the compression is adiabatic we can reduce (196) by means of (109), (129) and (135), as in art. 26, to the form

$$w_a = c(t_2 - t_1) + \frac{c}{n-1}\int_1^2 dt$$

$$\therefore\ w_a = \frac{cnt_1}{n-1}(r-1) \quad . \quad . \quad (197)$$

in which r has the value given in (186) and can be expressed by (135) in terms of temperatures, specific pressures or volumes.

2°. If the compression is isothermal we have, by (109)

$$pv = p_1 v_1 = p_2 v_2 = ct_1 \quad . \quad . \quad . \quad (198)$$

by means of which (196) reduces to

$$w_i = -ct_1\int_1^2 \frac{dv}{v} = ct \log_e \frac{v_1}{v_2} = \frac{-ct_1}{n-1}\log_e r \quad . \quad . \quad . \quad (199)$$

in which r has the value given in (186), but not that given in (135). The air after compression is conveyed in pipes to a distant point and there made to drive a piston.

3°. In case the air expands adiabatically, the total work done per unit of air in expanding from the state 3 to the state 4; when found by a process like that employed in obtaining (197) will be

$$w_{a'} = \frac{cnt_3}{n-1}(1-r') \quad . \quad . \quad . \quad (200)$$

in which t_3 is the temperature before expansion, and

$$r' = \left(\frac{v_3}{v_4}\right)^{1-n} \quad . \quad . \quad . \quad . \quad (201)$$

which can also be expressed as in (135), in terms of pressures or temperatures. Such an expansion can be practically effected only by heating the air before expansion, for by reason of the intense cold produced the vapor of water which is always found in air is congealed and so closes the valves, ports, etc.

4°. In case the expansion is isothermal, the total work performed will be

by (199), $$w_4' = ct_3 \log_e \frac{v_3}{v_4} = \frac{ct_3}{n-1} \log_e r' \quad . \quad . \quad (202)$$

in which the expansion is from the state 3 to 4, and r' is to be found from (201). In this case heat must be imparted to the air during the expansion.

5°. When the air is used without expansion and at full pressure, the total work performed per unit of air will be

$$w' = (p_3 - p_1)v_3 \quad . \quad . \quad (203)$$

38. Efficiency of Compressed Air.—Either adiabatic or isothermal compression may be employed, and then the air used in combination with adiabatic or isothermal expansion or with no expansion as in (203). The efficiency of these combinations are as follows:

1°. When adiabatic compression is combined with adiabatic expansion,

by (197) and (200) $$\varepsilon = \frac{t_3(1-r')}{t_1(r-1)} \quad . \quad (204)$$

in which t_3 the temperature just before expansion, would usually if no artificial means were employed heat the air, be nearly equal to to t_1 the initial temperature of the air. As before explained, such expansion is impracticable for air containing moisture.

If, however, heat be artificially supplied just before expansion to raise the temperature of the air from the normal temperature t_1 to the same temperature t_3, the amount supplied will be

$$k_p(t_3 - t_1) = \frac{cn}{n-1}(t_3 - t_1), \quad . \quad (205)$$

hence if this be regarded as part of the expenditure of energy the efficiency becomes

$$\varepsilon = \frac{t_3(1-r')}{t_1(r-1) + t_3 - t_1} \quad . \quad . \quad (206)$$

2°. When the compression is adiabatic and the expansion isothermal, heat must be supplied during expansion to an amount expressed, as in (182),

by $$t_3(e_3 - e_4) = \frac{cnt_3}{n-1} \log_e r' \quad . \quad (207)$$

which is the same as (202) the work done.

$$\therefore \varepsilon = \frac{t_3 \log_e r'}{t_1(r-1) + t_3 \log_e r'} \quad . \quad . \quad (208)$$

3°. In case the compression is adiabatic and the air is used at full pressure, the efficiency is found by dividing (203) by (197).

4°. When isothermal compression is combined with adiabatic expansion by (199), (200) and (205)

$$\varepsilon = \frac{nt_3(1-r')}{n(t_3 - t_1) - t_1 \log_e r} \quad . \quad . \quad (209)$$

when the air is artificially raised from t_1 to t_3 before expansion.

5°. When isothermal compression is combined with isothermal expansion by (199), (202) and (207)

$$\varepsilon = \frac{t_3 \log r'}{nt_3 \log_e r' - t_1 \log_e r} \quad . \quad (210)$$

6°. The efficiency of isothermal compression combined with no expansion is found by dividing (203) by (199).

When $t_3 = t_2 = t_1$, and $p_3 = p_2$, by help of (198) we find the efficiency in this case to be

$$\varepsilon = \frac{1 - \frac{p_1}{p_2}}{\log_e \frac{p_2}{p_1}} \quad . \quad . \quad . \quad . \quad (211)$$

The compressors in use permit a rise of temperature more or less great. The compression may be with sufficient correctness for practical purposes be taken to consist of two parts, of which the first part is adiabatic and involves the rise of temperature, and the second part is isothermal. The work performed in this case is found at once from (197) and (198). The circumstances under which compressed air is employed do not usually permit heat to be imparted to it just before or during expansion, hence it is used either at full pressure or with a comparatively small ratio of expansion. This expansion, owing to the conductivity

of the cylinder, etc., is not adiabatic, neither is it isothermal, but between the two. The heat thus derived from external sources contributes to the useful effect and renders that effect greater than expressed in (200). Zahner* has recently made a valuable practical discussion of this subject.

ERRATA.—In art. 15, write the equation following (26) thus,

$$\int_1^2 dh = \int_1^2 t\,de$$

In (130) put + for −.

In (144) put $\left(\frac{dv}{dt}\right)_p$ for $\left(\frac{dv}{dt}\right)$.

In (154) put $\left(\frac{dp}{dv}\right)_e$ for $\left(\frac{dp}{dv}\right)_e$.

Figs. 2, 3 and 4, which were accidentally omitted from arts. 15, 20 and 26 respectively, are here inserted.

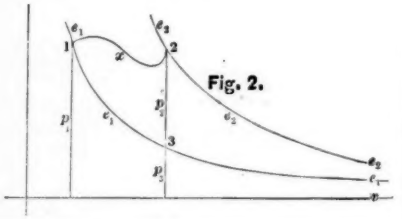


Fig. 2.

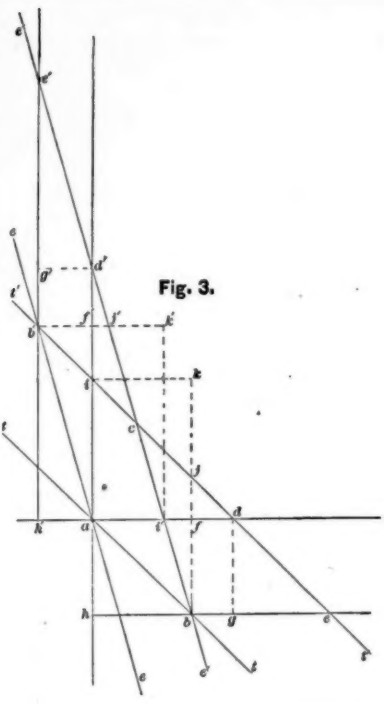


Fig. 3.

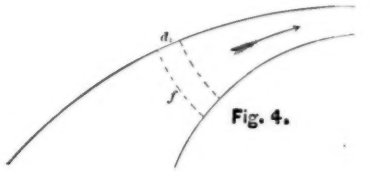


Fig. 4.

* Transmission of Compressed Air. Robert Zahner, M. E. New York: D. Van Nostrand. 1879.

SUBMARINE TELEPHONING.

BY CHAS. WARD RAYMOND, C.E., Junior of the Society.

Transactions of American Society of Civil Engineers.

A MEANS of communication, direct and at the same time easy and reliable, is and has been a great desideratum by those engaged in submarine work. The method of signaling now employed, by jerking upon the life-line, although, perhaps, sufficient for ordinary work, is, at the best, deficient in rapidity and reliability; and where the work is of such a nature that a word cannot express that which the diver would report to those above, it becomes necessary, in order to facilitate the work, that some other means be adopted.

Several attempts have been made in this direction, but none of them resulted satisfactorily. One may be mentioned which consisted in introducing into the air-supply circuit an air-tight chamber, sufficiently large to admit a person, com-

munication being kept up with the diver through the air-hose, by the assistant in the box or air-tight chamber.

It has been reserved for the telephone to solve the problem, and by the use of this instrument it has been already accomplished by several engineers, with favorable results.

Experiments were begun by the writer about September 1st, 1878,* to test the practicability and utility of the telephone in the submarine work in connection with the construction of the bulkhead wall, now being built by the Department of Docks of this city.

The instruments used at first were two "Phelps' Duplex" telephones, loaned for the experiments by the Gold and Stock Telegraph Company, through the courtesy of Geo. B. Scott, Esq., Supt.

The "Phelps" telephone is peculiarly adapted for this purpose on account of its shape and size. It is oval and flat, 5 inches by $2\frac{1}{2}$ inches, and $\frac{3}{4}$ inch in thickness (12.7x6.35x1.59 c. m.).

One telephone was placed in the diver's helmet, and fastened in such a position that, by simply turning his head, the diver could place his mouth or his ear to the instrument. The other telephone was placed on the scow which carried the air-pump and diver's helpers.

The connecting wires were insulated (double-covered paraffined office wire, No. 18) and passed through the air-hose.

Beginning at the helmet telephone, the two wires pass from it through a small opening made in the fan-shaped air-distributor in the back of the helmet, the hole being bushed, to prevent wearing the covering of the wires; passing into the hose, and through the hose until they reach the couplings at which connection is made with the upper telephone. Two couplings, separated by about one foot of hose, are inserted in the hose, at a convenient distance from the air-pump, in this case about 15 feet. To the inside of each coupling is soldered a copper wire, to which the insulated wires from the helmet are fastened. A binding screw is fastened to the outside of each coupling into which the wires from the upper telephone are inserted, thus completing the circuit.

*The idea was conceived in March, 1878, but opportunity to carry out the experiments was not afforded until September.

This was found to work very well so far as communication from the diver to his helper was concerned, but in the contrary direction it was by no means satisfactory, as it was found necessary by the diver, when he wanted to hear, to stop the escape of air from the helmet into the water, which bubbled so as to interfere with his hearing.

This stopping the escape of air is done by the diver stooping over, thereby allowing the air to fill the diver's dress, and, of course, affords only temporary relief.

To remedy this as far as possible, a cloth tube, about 6 inches in length, and of about the same diameter as the escape valve-cap, was fastened by one end over the escape, leaving the other end free. This gave better results, but still not satisfactory.

It was then decided to try an "Edison Carbon Transmitter," which is a form of microphone, and intensifies, or, more properly, increases the volume of the sound. This necessitated the addition of an induction coil and one cell of battery; in this case a "Gravity" battery was used, the arrangement of wires and telephones remaining the same.

The action of the carbon transmitter is quite simple, and on the principle already established, that the secondary or induced current possesses a high degree of intensity, this new arrangement resulted in the perfect success of the experiment. Conversation was carried on with the utmost facility; it was not necessary to give the diver any signal other than a simple "hallo!" It was also found that the diver could talk in the helmet without putting his mouth to the instrument and be heard plainly, and therefore he could continue his work and conversation at the same time.

The battery, induction coil and transmitter were placed on a shelf on the diver's scow, and together occupied no more room than would a Webster's Unabridged Dictionary; the telephone in the helmet occupied but little room, and, of course, was not at all in the way.

The practicability of the telephone for this purpose was thus definitely settled.

These experiments were conducted in depths varying from 0 to 30 feet without any perceptible variation in effect.

It has been reported to the writer that

at a depth of 50 feet the telephone failed. I do not know what the conditions were, but, should any difficulty be met with at that or at greater depths, it can be easily remedied by using a second transmitter, placing it in the helmet on the opposite side from the telephone, and connecting it with its own battery and induction coil—placed on deck—by two extra wires running through the hose to two extra couplings. These four wires may be small in size and occupy no more space in the hose than did the two previously used.

I feel safe in asserting that telephones thus arranged can be used at as great a depth as a diver can work, if not at a greater one.

It may not be out of place here to mention a fact, as curious as it is novel, which was told the writer by Messrs. F. Collingwood and W. H. Paine, Assistant Engineers of the New York and Brooklyn Bridge, that attempts to signal by whistling, with the mouth or fingers, were a decided failure in the caisson when at a depth of about 44 feet. This may possibly be explained on the supposition that, under a pressure of $2\frac{1}{10}$ atmospheres, it might be necessary to change the relative position of the fingers and tongue to produce the sound, which practice in the ordinary condition of the atmosphere enables us to produce. And, also, that in whistling in a dense medium a resistance is met with which is not favorable to the production of the sound, but which, when produced, is conducted with greater rapidity, and consequently is heard more distinctly. From this it may safely be concluded that where the diver can speak the telephone will transmit the sound. As to his being able to hear at that depth there is no doubt.

As regards the utility of the telephone in submarine works, so short a time has elapsed since its introduction into this Department that no conclusions can be drawn other than would naturally present themselves to the intelligent mind.

Much, of course, depends on the nature of the works upon which the diver is employed. In extended general examinations, where the diver has much to report, and where any delay would be inexpedient, its utility would best be appreciated.

Mr. Francis H. Fisher, Assistant Engineer to Mr. Edward S. Philbrick, on the heavy sea wall, in Boston harbor, has kindly furnished me with the results of his employment of the telephone.

He made use, finally, of but one wire, which was wound spirally around the outside of the air-hose, and both hose and wire then wrapped with canvas, to protect the insulated wire from injury. The circuit was completed by attaching one wire to the helmet and the other end to a copper plate suspended in the water from the bottom of the scow. A slight objection to this arrangement may be that communication can only be kept up with the diver so long as the helmet is in the water, and that where the wire passes from the outside through the helmet, the opening must be insulated and air-tight; the wire is apt to become loose and its insulation destroyed. I think the method, herein described, of passing the wire through the hose, much the better way. In this case, two wires form the circuit, are entirely out of the way, are protected by the hose itself, and no other insulation than being covered is necessary. The circuit is thus always complete, whether the helmet is under or above water, as frequently happens in shallow water.

Mr. Fisher testifies to the utility of the telephone in saying: "The necessity of using the telephone arose in providing for a thorough examination of the masonry in the face of the sea wall, below water, in which it was necessary to record the dimensions and positions of the joints around each stone for some courses throughout the wall. The accomplishment of this would have been impracticable, if not impossible, had the diver been obliged to come up to report; but with the telephone it was made in every way successful." Mr. Fisher used a compact form of the Bell Telephone.

From these and other experiments, we believe that submarine operations can be relieved of much expense, both of time and labor, by the use of the telephone. And much submarine work, heretofore considered impracticable, can now be carried on with facility and expedition. In the caisson and in the mine its value would be great, especially so in the latter, where, should one portion of the mine, by an accident, be shut off from the rest, communication by the telephone might be preserved.

THE TURBINE WHEEL DISCUSSION.

A REPLY TO PROF. BURR'S ARTICLE.

By Prof. W. P. TROWBRIDGE, Columbia College.

Written for Van Nostrand's Engineering Magazine.

Professor Burr's criticisms of my article on Turbine Wheels in the last number of your Magazine are prompted, doubtless, by a desire on his part to remove the "impressions of a very erroneous character," which, according to him, my article is "likely to produce."

In his discussion he assumes that I have misunderstood the investigations of Bresse, Rankine and Weisbach, and accordingly supplies his own interpretations of the views held by these authors on the question of impact. At the same time misinterpreting my article on the same subject, in such a way that, like Buttercup in the play, he "mixes us all up."

He intimates that I regard shock as of no importance, while the authors referred to, although "they said" it didn't mean it; or, to speak more plainly, although they "say" that impulse must be avoided, yet in their final results it is quite ignored.

Although Professor Burr's statements on this matter are somewhat confused, yet when carefully scrutinized they amount to this, when taken in connection with the matters of fact to which they relate.

That when the *conditions* of a mechanical problem are laid down and expressed mathematically, the result of combining the mathematical expressions may be and is in this case to eliminate the *conditions* on which they were founded from the final result. A remarkable proposition, new to mathematics, new to mechanics, in fact, new to science.

That Professor Burr does actually maintain such a proposition in his paper (unconsciously of course), and considers it the lever by which he overturns my article, I propose to demonstrate.

Professor Burr admits that the authors, above mentioned, lay down a certain mechanical axiom as a condition essential to the best construction and working of a Turbine wheel; but he says that in their final expressions for efficiency there is nothing whatever depending on such axiom or mechanical condition. This question is one of facts, not of opinions, and I shall show that these authors do lay down such an axiom, as stated in my paper; but as to the last assertion that that condition is finally eliminated the facts are against Professor Burr.

Indeed it might be at once taken for granted that such eminent mathematicians, as those above mentioned, would not lay such stress upon a particular mechanical condition as to insert it as a mathematical condition, and then proceed to eliminate it altogether. I propose to show that they were not guilty of any such folly.

I will first make quotations from Prof. Burr's paper for reference, and then endeavor to point out the fallacies of his deductions.

He states that my "objection seems to be that Weisbach, Rankine and others 'insist' that for the best performance of a wheel the water shall enter without shock:" while I hold "that an equal efficiency, at least, may be attained if the relative velocity be not tangent to the buckets at the point of entrance to the wheel." I do not "object" to the authors named insisting on the principle that the water must enter the wheel without "shock." On the contrary, in discussing the *reaction* wheel, I show that this is as it should be an *essential feature* of the *Fourneyron reaction wheel.* What I do endeavor to demonstrate is that this leading axiom, which all of the authors named impose in advance upon their discussions *limits these discussions* to the *reaction* wheel.

In regard to the last part of the paragraph, "while he (I) holds that an equal efficiency, at least, may be obtained, if the relative velocity be *not tangential* to the buckets at the point of entrance to the wheel. In other words, if the resistances be left out of consideration, it is a matter of no importance whether there is or is not *shock.*" Professor Burr must have

read my article hastily or he would not have received this "erroneous impression."

He will find, I think, that so far from regarding shock as a matter of "no importance," it is one of the chief causes of efficiency and work in Impulse-and-Reaction Wheels. Shock, impact and impulse, being in the sense in which we are using these words synonymous. That these words are synonymous in the discussions of Bresse, Weisbach and Rankine I will presently show.

But I will follow Professor Burr a little further, first, on the same subject.

He says: "Now if the consideration of resistances be omitted, and if shock is of no consequence, then it follows that a formula for curved floats ought to give precisely the same result whether the relative velocity of entrance be tangential in direction or not." "In fact the formulae of Weisbach show this, as do also those of Bresse and Rankine." "In order to verify this statement let any one turn to the expressions for the efficiency of a turbine in art. 260 of Du Bois's translation of Weisbach's work on water wheels, eq. 21 of art. 15 of Mahan's translation of Bresse on Hydraulic Motors, and equation 6, art. 174 of Rankine's "Steam Engine and other Prime Movers."

"*In no one of these expressions is there anything found depending on the angle which the bucket makes with the circumference of the wheel at the point of entrance.*" *Now the matter of shock or no shock depends upon that angle, and since the expressions for the efficiency do not in any wise depend upon such a value they manifestly are* NOT *based upon the theorem or axiom that the "water must enter the wheel without shock."*

I have made this long quotation in order to allow the authors mentioned to speak for themselves afterwards as to their meaning, and especially as to the statement that "the matter of shock or no shock depends upon the angle which the bucket makes with the circumference to the wheel at the point of entrance".

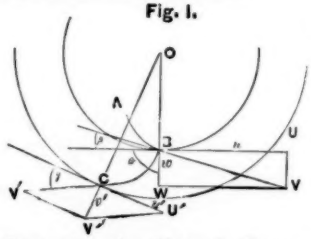
Rankine says: "*In order that the water may work to the best advantage it must enter the wheel without shock and leave it without velocity; for which purpose the velocity of whirl on first entering the wheel must be equal to that of the first circumference of the wheel,*" &c. Here Rankine says nothing about the angle of the blades, but he does lay down the axiom that the water must in no case have an impulsive effect upon the wheel. Again, page 196: "*The greatest efficiency without friction is altered, as has been stated, when* $v = ar$. (Heading of art. 175, Rankine.) In this quotation v = the tangential component of the velocity of the water and $\overline{ar}$ the tangential velocity of the wheel where the water impinges upon it.

No reference is here made to the angle formed by the blades with this circumference, but the condition above quoted is made one which must be imposed for the maximum efficiency.

Let us now refer to Bresse.

The following figure is copied from Mahan's translation of Bresse:

Fig. I.



O is the center of motion of a Fourneyron wheel. AB a guide blade. BC a bucket, BU = u the velocity of the inner circumference of the wheel at B, and BV. = v the velocity of the entering water. BW = w is the relative velocity of entrance to the wheel, CW' = w' is the velocity of exit v' the absolute velocity of exit or the lost velocity.

In establishing the equations from which, with other equations, *his expressions for the efficiency is derived*, Bresse makes the following statement, (see Mahan's translation page 82). "*There still remains to express two conditions for obtaining the best effective delivery. It is necessary that at the point* B (see sketch) *that w* (*i.e., the relative velocity*) at the point of entrance be directed *tangentially to the floats*," and this "axiom" or primary condition of the problem is introduced mathematically by the equation

* Rankine "Steam Engine and other Prime Movers," page 192.

$$\frac{n}{v}=\frac{\sin(\theta+\beta)}{\sin\beta}.$$

(see sketch for the angles θ and β).

This equation of condition is simply that of the sines of the angles being proportional to the opposite sides.

Bresse thus obtains the first of his two "remaining conditions," which is precisely the same as that of Rankine above quoted. It will be seen from the sketch which I have given, as well as from the above quotations, that the question of shock or no shock does not depend upon the angle of the blades alone, but upon the condition mentioned by Rankine, that the wheel velocity and the component of the water velocity in the same direction shall be equal.

For Weisbach's view of this matter we may refer to DuBois' translation page 399. In laying down the theory of the *Reaction Turbine* one of the first conditions named is as follows:

"*In order that the water may enter the wheel without impact this velocity v* (*i.e.*, the velocity with which the water leaves the guide blades) *must be decomposed into two others, of which the first must coincide with the inner velocity u* (*the velocity of the inner circumference of the wheel*) *and the other w having the direction of the* entering *water in the wheel*," we have therefore for the last,

$$w^2=v^2+u^2-2uv\cos\beta.$$

(The notation in Weisbach has been changed to correspond with that of Bresse).

Professor Burr states that the question of shock or no shock depends upon the angle made by the blades with the circumference at the point of entrance of the water, and that *angle* being *eliminated*, the condition itself of shock or no shock disappears from the final result.

The absurdity of the assumption will appear in a stronger light, when the expressions for efficiency are analyzed to which he refers your readers.

That of Bresse is

$$\varepsilon=1-\frac{V'^2}{2gH}.$$

This expression it will be seen contains the expression for the lost velocity V', which depends directly upon the *relative velocity of entrance w, and the latter depends directly upon the angle*, which Professor Burr says, does not appear in the expression for the efficiency. It is true that it does not appear in the sense that its symbol can be seen, but nevertheless it is there involved, and will not "out" at any one's bidding, and it is mathematically absurd to say that its influence has been eliminated.

Professor Burr cannot dispute these facts if he will read more attentively what Bresse and Weisbach have to say on the subject.

Weisbach expressly states that "*the exit velocity* upon which the *lost velocity directly depends, depends on the entrance velocity*, and Bresse states expressly that the angle β to which Professor Burr alludes should not be *too acute because it would increase the entrance velocity.*

Professor Burr is thus absolutely in the wrong when he states, that the question of "shock or no shock" depends only upon *an angle*, which is eliminated from the expression for efficiency. The question depends upon other elements of the problem, and especially upon the relative velocity of the wheel and water, and indeed if the angle referred to and the direction of the entering water to be fixed in advance, the question depends upon the equality between the wheel velocity and the component velocity of the entering water.

By reference to facts I have shown that he is quite as much in the wrong in asserting that the expression for the efficiency is independent of the angle to which he so often refers.

In regard to the expressions for the efficiency of turbine wheels, Professor Burr lays much stress upon the identity of my formula with those of Bresse and Rankine (when frictional resistances are left out of consideration). He cannot be ignorant of the fact that these formulas have nothing to do with the theory of turbine wheels as far as *form* is concerned, any more than they are the formulas for efficiency of the steam engine or any other motor employing energy. The universal formula for efficiency of all motors is $\varepsilon=1-\frac{W'}{W}$; W' representing the energy lost, and W being the whole disposable energy.

In the case of heat engines this takes the form

$$E=1-\frac{T_1}{T_2}$$

In the case of a fly-wheel doing work

$$E=1-\frac{V_2^2}{V_2^2}$$

In the case of water acting by its weight

$$E=1-\frac{h_2}{h_1}$$

In the case of a propeller

$$E=1-\frac{S}{S'}, \text{ \&c., \&c.}$$

The efficiency depending, with a given quantity of disposable energy, upon imposing such *conditions* upon the motor that the energy lost, represented by the numerator in the second member, shall be a minimum.

In my paper on Turbines, I endeavored to show that Bresse, Weisbach and Rankine have imposed such conditions upon the motor which they discuss as to make it a *reaction wheel*, and that these conditions do not apply to the best modern wheels, introduced by Boyden & Francis, because the latter are *impulse*-and-reaction wheels, not restricted by such conditions.

I gave for the first time, as far as I am aware, the theory of such wheels, and stated that if Boyden & Francis had followed the rules laid down by the authors above quoted, they would not have improved the old Fourneyron wheel, and would not have produced the inward flow wheels, which are now becoming unusual.

In regard to the rules laid down by Bresse and Weisbach, applying as they do to purely reaction wheels, they advise that the entrance angle of the water should be nearly 45°.

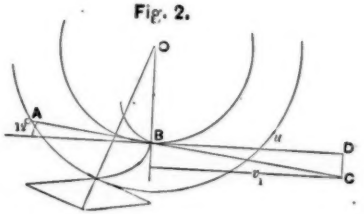
Bresse says, page 92 (Mahan's Translation), the sum $2\beta+\theta$ should be nearly 180°, and θ should be nearly 90°. This makes β as the best angle 45°.

Weisbach, in applying his "arrangement," referred to by Professor Burr as "an admirable arrangement," says that the entrance angle should be about 55° for the best efficiency, but, because the guide blades would then be too divergent, he thinks it better to make the entrance angle smaller.

Now, in some of the best modern wheels this angle β, or a of Weisbach, is made as small as 12°.

Let us see what would be the effect, with this small angle, of enforcing the condition that the "velocity" of the wheel must be equal to the component of the velocity of the water in the same direction. The following sketch will illustrate the construction:

Fig. 2.



Let v_1 represent the velocity of the water entering at an angle of 12°, n the velocity of the wheel, and also the component of the velocity of the water in the same direction. There is no effect from impact or impulse, and the water enters the wheel with the small relative velocity DC, and this is all the available energy with reference to this wheel. In order that it may leave the wheel with a relative velocity equal to the second circumference, its velocity must be increased by the centrifugal effect of the convex surface of the blade, and the wheel would actually have to *pump out* the water. With this small angle the whole energy of the water is made effective if it be allowed to act by impulse, or, in other words, if the velocity of the wheel at its inner circumference be less, in proper proportion, than the component velocity of the entering water. And this is the law of the Boyden & Francis wheels; a direct violation of the axioms or mathematical conditions imposed by the authors who have so often been referred to.

In regard to the results which Professor Burr obtains from using Weisbach's formula for the efficacy for Francis' experiments, he must use certain coefficients, and if he will publish those coefficients and state by what experiments they were derived, as especially applicable to Turbine wheels, he will furnish information which I for one would be glad to obtain. As far as Weisbach's own ex-

amples furnish data, I consider the coefficients mere guess work; there is one thing especially apparent in that author's valuable work on the dynamics of fluids; viz., that coefficients of resistance under varying conditions are exceedingly variable, and when such coefficients are applied to unknown conditions, or new conditions, they must be taken with great allowance, in fact are merely approximate guesses.

Now it was shown by Francis in his experiments that a *variation of velocity of forty per cent. produced a variation of efficiency of two per cent. only.*

The extreme accuracy of Professor Burr's deductions is, therefore, suspicious. Not that Professor Burr would purposely cook the results: I would emphatically disclaim making such an imputation, but that other results could be obtained with as good a show of general accuracy in the use of coefficients I have no doubt.

I suppose Professor Burr is aware of the fact that Weisbach rather contemptuously doubted the correctness of Francis' experiments, in obtaining some of his large efficiencies, and that author surely ought to be a good judge of the applications of his own formulas.

Professor Burr lays much stress on the "admirable arrangement" of Weisbach to prevent "leakage," as Weisbach expresses it, between the wheel and the case.

This "admirable arrangement" consists, for wheels whose buckets are radial, simply in making the guide blades at such an angle that the water enters the wheel at an angle of 45° with the circumference. (See Du Bois' translation of Weisbach, page 405.)

Would Professor Burr recommend all Turbine wheel manufacturers to adopt this "admirable arrangement" in their constructions? I think not; I hardly think his advice would be adopted, at all events, if he should.

Finally, I must correct Professor Burr in another important point. He states that I regard it as a matter of indifference whether there be shock or not. He has become somewhat confused as to what the authors to whom he clings regard as shock, and I have shown conclusively their own interpretation of it, viz. *impulse.* In this sense my paper shows that I regard shock as of very great importance in wheels as they are now almost universally made and run.

What I did say and distinctly conveyed in language that no one should misinterpret was, that undue importance had been attributed to shock or impulse as *to its effects in producing the disturbances which it was the object of the old authors to avoid* by their restrictions. That in limiting their wheels to a particular class, and the best wheels even to an exceptional case of a class, they had, by their rules, shut out the kind of practice now in vogue. That the disturbances which they feared were not so great as had been supposed.

The practical confirmation of my ideas in this respect may be found in the performances of the best wheels now in use, in which this dreaded but very useful shock exists with all its attendant *disadvantages* as well as advantages.

If Professor Burr should still want further instructive evidence of the efficacy of impulse, and of the distinction which Rankine and Weisbach make between impulse and reaction, I would refer him to those pages in the works of these authors which treat of the *impulse* and *reaction* of water upon *vanes.*

In my paper I extended their theories to *Impulse* and *Reaction Turbines,* while they confined their discussions of turbines to *Reaction* Turbines. I was prompted to do this because I felt that their discussions were not *general enough* to include the best practice of the present day.

I will close this reply, by quoting the well deserved tribute to Boyden and Francis, of an eminent hydraulic engineer, a Frenchman by birth, given in a recent public lecture. He says:

"The Turbine on the Screen is an exact copy of one built by Fourneyron himself forty years ago. * * * Were I able to give you on the same screen the same class of turbines as constructed by Boyden and Francis, you could not avoid admiration for the thoroughness of their researches in finding, applying, and perfecting curves, and for the mechanical appliances whereby the water is divided and prepared, from which curves and appliances were derived those astounding results of the total efficiency of the water power."

THE METRIC SYSTEM.

REPORT OF THE STANDING COMMITTEE ON THE METRIC SYSTEM OF WEIGHTS AND MEASURES, AND ACCOMPANYING LETTERS, PRESENTED AT THE ANNUAL MEETING OF THE BOSTON SOCIETY OF CIVIL ENGINEERS, MARCH 19, 1879.

To the Boston Society of Civil Engineers:

The standing committee on the Metric System of Weights and Measures request your careful consideration of two subjects:

The first is the familiar theme of petitioning Congress. Petitions were sent to the last (the 45th) Congress by the following organizations (and possibly others) besides our own society:

New Jersey State Medical Society; New Haven Engineering Society; Cincinnati Society of Natural History; Medical and Chirurgical Faculty of Maryland; Society of Arts of the Massachusetts Institute of Technology; Boston Society of Medical Sciences; Boston Society for Medical Observation.

The Ohio Association of College Officers also voted to memorialize Congress.

The tenor of their memorials was not precisely the same, but they generally prayed for the exclusive use of the metric system in various branches of the Government service after some future date. The New Haven Engineering Society specified the Post-Office, the Custom-House, the Mint, and the Engineer Corps. A week ago, in New York, the United States Board of Trade voted that the Government be recommended to put the metric system into practice in the Post-Office and Custom House.

The first session of the 45th Congress opened 15th October, 1877. Our last written report to this Society was presented 17th October, 1877; and the Society that evening voted to memorialize Congress for the passage of a resolution inquiring of the heads of the Executive Departments what objections there are to the adoption of the metric system in the Government business and in private transactions, and how long a time should be allowed for such adoption, the same resolve, in short, which had been proposed in the preceding Congress by Hon. Milton Sayler, of Ohio, but had not been acted upon. Mr. Sayler presented our memorial 3d November, 1877, and had it printed in full in the Congressional Record. It named seventeen organizations from whom we had received assurances of co-operation; among them were the Engineers' Club of St. Louis, and the New England Association of Gas Engineers. The resolution as prayed for was promptly passed by the House of Representatives 6th November, 1877, and replies thereto were made about a year ago by twenty-four Government officers of various departments and bureaus. Some presented elaborate reports; some brief and specific answers to the inquiry; a few stated that having nothing to do with weights and measures they were not in a position to judge. As a single illustration of the variety of opinion that has been brought out it may be mentioned that the Surgeon-General of the Army thinks the adoption of the metric system would be fraught with grave danger to the sick soldiers; while, on the other hand, the sick sailors are now being dosed according to it, the medical purveyor's department of the Marine Hospital Service having actually introduced it last year; and it is believed that the health of the Navy has not suffered in consequence.

The House Committee on Coinage, Weights and Measures having digested these various reports, has printed them and many other documents in a considerable volume, along with its own report presented 7th January, 1879. A copy is to-day added to the library of the Society. In a speech of Hon. Levi Maish, of Pennsylvania, printed in the Congressional Record for 8th March, 1879, occur the following sentences: "I introduced a bill in Congress making the use of the metric system obligatory in the custom-houses and post-offices of the country. The committee, however, thought this would be too great a stride to make at once toward the adoption of the new system."

Nevertheless, the report just now mentioned earnestly recommends the early passage of Mr. Maish's said bill.

We have repeatedly expressed the opinion that it rests with Congress to take action upon the metric system. A deliberate recommendation has now been made by a standing committee of eleven members headed by the venerable Alexander H. Stephens, after a full presentation of the facts; and it was voted that 10,000 copies should be printed; we think their recommendation should be acted upon by this Society. Moreover, as it is very evident that the complete introduction of the metric system can only be accomplished by general concurrence, it is manifestly reasonable to begin by asking the largest user of weights and measures, namely, the United States Government, to join in the movement.

We submit the following draft of a new memorial:

To the Hon. the Senate and the House of Representatives of the United States in Congress assembled:

The Boston Society of Civil Engineers respectfully prays that in accordance with the earnest recommendation of the Committee on Coinage, Weights and Measures of the House of Representatives, as expressed in Report No. 53, 45th Congress, 3d Session (p. 34), it may be enacted as follows:

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled,

Sec. 1. That on and after July first, eighteen hundred and eighty-one, for all postal purposes, fifteen grams shall be substituted for half an ounce, and so on in progression.

Sec. 2. That on or before July first, eighteen hundred and eighty-one, the Postmaster-General shall furnish all post-offices with postal balances denominated in grams of the metric system, at an expense not exceeding fifty thousand dollars, which sum is hereby appropriated, or so much thereof as may be necessary for this purpose.

Sec. 3. That on and after July first, eighteen hundred and eighty-one, the metric system of weights and measures, as legalized in section thirty-five hundred and sixty-nine of the *Revised Statutes*, shall be obligatory in the assessment of duties on imported commodities in the custom-houses of the United States.

Section 3,569 of the *Revised Statutes*, is the law of July, 1866, establishing the equivalents of the metric units in terms of the customary weights and measures.

The form of law proposed is the same as Mr. Maish's bill, with the exception that we have altered by one year the date for it to go into effect, because of the delay of a year in the time of its expected passage.

The 46th Congress opened yesterday, and we respectfully urge upon this society that at the earliest reasonable moment it be

Voted, that the memorial submitted 19th March, 1879, by the Committee on the Metric System, be adopted by the Society, signed by the President and Secretary, and transmitted to both Houses of Congress.

The desired legislation will be made when it is demanded by our citizens; it is hoped that the demand will be prompt and general. The undecided and conservative people who are anxious to proceed very cautiously, and take extremely slow and gradual steps, ought to join in this appeal. Nearly all can agree upon this first step. We don't mean, however, to disguise our own opinion, as stated heretofore, that a simultaneous movement is required for the most economical conduct of the reform; that the establishment of the metric system in the Post Office and the Custom House should be rapidly followed, say in five years, by its exclusive use in all other branches of the public service; and that the people of the United States (and particularly those persons who petition Congress for the change) ought to adopt the metric system in their private affairs directly after beginning to use it in their dealings with the Government. The more quickly the transformation can be completed, the less protracted will the annoyance be; provided, of course, that it be not attempted to accomplish it within shorter limits than are reasonably fixed by the requirements of any one ordinary business. In this connection it may be interesting to quote the opinion of Gen. Humphreys, Chief of Engineers. He says, 3d December, 1877:

"So far as the proposed change would affect the works carried on under charge

of the officers of the Corps of Engineers, it need only be said that while any change in the ordinary and accepted standards must be an inconvenience, yet there is no other reason why the change should not be made, provided sufficient time is given for preparation. It is thought that the French metric standards should not be adopted to the exclusion of the present standards, in this office within a less interval of time than five years after the passage of the act. This limit is fixed as the minimum, in order to allow for the proper careful manufacture, comparison, and distribution of standards, and their duplication in various forms for ordinary use, for the necessary changes in tables and formulæ, and more especially to allow a sufficient interval of time during which a practical familiarity with the new standards may be acquired, particularly by those with whom the business of the Engineer Department is transacted and who are not in the public service, as well as by those not in the public service who use the maps, charts, etc., of the department."

The practical steps toward the actual use of the meter in our own profession will be the topic of the remainder of this report. The little matter of plotting a metric scale upon those plans that have any linear scale has formerly been spoken of as the beginning. The disposition of engineers toward that preliminary measure is illustrated by a letter (a copy accompanies this report) sent last October to the Library Committee of the American Society of Civil Engineers, and by the fact that they purpose, hereafter, to put a metric scale upon the illustrations of papers published in their Transactions, where no objection is made by the author. The Engineers' Club of Philadelphia adopted, April 6, 1878, the report of its Committee on Metric System, which requested "that, in all papers read before the Club, the Metric System be used conjointly with the English, and that, on all maps, sections, and drawings, a metric scale be placed, for comparison with the ordinary mile, foot, or inch scale." They also recommended the introduction of the metric system into machine shops, public schools, and professional literature. The report was printed and widely circulated; copies were courteously sent us, and were submitted to the Society last spring, as may be remembered. The Civil Engineers' Club of the Northwest, devoted a meeting to the metric system, November 6, 1878. The arguments in its favor were very ably presented; and a motion was carried instructing the Secretary to procure for the use of the Club, stationery printed with metric devices. It was stated in the discussion, that on the U. S. Lake Survey the metric scale is being attached to charts. A full report may be found in *Engineering News*, for November 14 and 21, 1878. A final report by the committee appointed three years ago, and decisive action by the Club, have yet to be made.

The American Society of Civil Engineers, by letter ballot, canvassed February 6, 1878, "Resolved, that the further consideration of the metric system of weights and measures be indefinitely postponed," 102 to 57.

We think ourselves that the further consideration of the advisability of adopting the metric system may be superfluous, regarding its ultimate use all over the world as a foregone conclusion; but we believe it is worth while to study the best method of introducing it. We look forward to a time, not far off, when there shall be a general movement for the actual use of the metric measures and the abandonment of the old ones. The first effective substitution in the work of our own profession may be expected in surveys of land. Our second subject, therefore, which we wish to examine at considerable length, is the metric system in the measurement of land; for, in fact, its use there has already begun.

The meter has always been the standard of the U. S. Coast Survey, to take the most conspicuous instance; though, strange to say, its published charts bear linear scales of miles, but not of meters, and their ratios to natural size have sometimes been $\frac{1}{15000}$, $\frac{1}{30000}$ and others that can not be expressed precisely as decimal fractions. During the past two years a re-survey of Boston Upper Harbor has been made in meters, under the direction of the Massachusetts Harbor Commissioners; their last report gives a list of geographical positions of land marks and conspicuous objects with which local surveys can readily be connected, distances being

given in meters only; it contains also a skeleton map of the triangulation bearing a linear scale of meters alone. Passing from large surveys to small ones, Mr. Charles W. Howland, of Rockland, Mass., has been making surveys in meters and corresponding plots for deeds for his patrons in Rockland and neighboring towns, during the past five years or more; and he finds that it works perfectly well.

The reason for beginning the use of the metric system upon this class of work rather than upon any other that falls within the province of the engineer is that it is the least connected with other people's measurements. There is a considerable amount of land surveying of this class; surveys merely for the measurement of area, or for making descriptions and plans for deeds of record, or for dividing a parcel of land by lines fulfilling given geometrical conditions, are nearly independent of other measurements, and the surveyor can use what unit he likes. Whenever he shall become convinced that the speedy adoption of the metric system is assured, he will be able to use the metric measures upon work of this class more readily than elsewhere. Measurements of land, moreover, unlike those of the warehouse and the workshop, are liable to be referred to for many years. The plans that we are making to-day may be preserved in our Registries of Deeds, and be appealed to a century hence. According to general opinion the metric system will then be in use everywhere, and if the coming surveyor shall not find it on our plans, he will suffer a similar inconvenience to that which we who use feet now endure from the chains, rods and links of a former generation.

The introduction of the meter will require but very little effort. Metric chains, tapes, poles and rules cost no more than those of the old-fashioned style. The lion which most of us see in the way is probably the incongruity between the meter and the foot. What we ought to aim at in making the change will be to use the meter only and not to refer to the foot at all; the incongruity will then be a help, not a hindrance. Nevertheless, the interconversion of old and new measurements will occasionly be required at the best; and to people who are on the lookout for obstacles, that prospect is rather formidable; but when we look upon it as an inevitable task which we must contrive to perform in the easiest way we can, we shall see that it is the same sort of thing that we are already (to our sorrow) well acquainted with in the transfer of miles, chains, rods, and links into feet, and in the change of inches and sixteenths into feet and decimals. The change into meters is doubtless of somewhat greater magnitude; but it is not so much greater as might at first sight be supposed; and by an adroit selection of examples, may even be made to appear less. We sometimes see a long string of decimals (.025399772) written to express the value of an inch in meters; to express the value of an inch in *feet* takes just as many decimals, .083333333+, and is not mathematically exact at that. Suppose that we come down to reasonable figures for practical use; for instance, a surveyor who uses decimals of a foot is called upon to give lines or grades, for masons and carpenters, who are building from architect's plan figured in inches; the surveyor is exact enough if he uses three places of figures, and calls 5 inches 0.417 of a foot, and 10 inches 0.833 of a foot; that is, he neglects one part in 1,250, or one part in 2,500; if, on the other hand, he had used a metric tape and three places of decimals, and had called 5 inches 127 millimeters, and 10 inches 254 millimeters, he would have made an error of less than one part in 100,000. To reduce rods to feet you must multiply by 16.5, miles to feet by 5,280, still three significant figures. Take square measure; to reduce acres to square feet you must multiply by 43,560, four significant figures; to reduce square rods to square feet by 272.25, five significant figures; and even to reduce square feet to square inches, you must multiply by 144, three significant figures. On the other hand to reduce square feet to square centimeters, you must multiply by 929; square feet to square meters by .0929; square feet to hektars, by .00000929; which is three significant figures. The computation is made a simple matter by the aid of tables, some of which are very conveniently arranged. A new table by Mr. Emonts, of the American Society of Civil Engineers, will probably be printed soon. For approxi-

mate reckoning which will be sufficient in many cases, a graphical method can be applied. (See accompanying diagram.)

As another illustration of the readiness with which the change can be made, take the matter of draughting on plans. Suppose a surveyor makes a drawing of a building lot on a scale of 10 feet to an inch, that is, $\frac{1}{120}$ or $8\frac{1}{3}$ per thousand of full size; that plan is sent to an architect's office to guide him in making his designs; suppose he wishes to read distances upon it in feet and inches, the nearest scale he can find to it on the common architect's triangular draughting scale is $\frac{3}{32}$ of an inch to the foot, which is $\frac{1}{128}$ of full size; if he should use that scale, therefore, $\frac{1}{16}$ part or $6\frac{1}{4}$ per cent. would have to be deducted from each reading. Suppose, on the contrary, that he should wish to scale distances in meters from the plan; the nearest standard decimal scale would be .008, or 8 millimeters to a meter; and if this were used, $\frac{1}{25}$ part or 4 per cent. would have to be deducted from each reading would have to be deducted from each reading to get the measurement in meters. As another instance, suppose you have to add some work to an old plan on a scale of 100 rods to an inch. To plot feet on it you require a scale of 1,650 feet to an inch, which is not ordinarily to be found engraved ready for use; it is $\frac{1}{1980}$ of nature, that is, one per cent. larger than $\frac{1}{2000}$, and, as paper plans are liable to shrink as much as that (and plans printed on damp paper considerably more than that), a metric scale of .00005, or 5 centimeters to a kilometer, can be applied to plot metric measurements as conveniently as anything. Similarly scales of 20 rods to an inch, 5 inches to the mile, and $2\frac{1}{2}$ inches to the mile vary about one per cent. from. 00025, .00008, and .00004.

It is not pretended that these are average cases; they are confessedly selected for a purpose; but to any mind familiar with this matter, examples enough of a contrary tendency will undoubtedly occur to secure a fair judgment. The present discussion is simply intended to give a realizing sense of the fact that the labor of translating quantities into the metric system need not alarm engineers, who can at one time use miles and acres on extensive land surveys, feet and decimals on railroads and building lots, cubic yards on earthwork and masonry, and inches and sixteenths on wooden and iron construction.

By the adoption of the metric system, several material advantages are to be gained in the details of the surveyor's work, which are often overlooked in our general view of its more important and extended benefits.

Apply the metric system to draughting.

1. We shall use the same scales that architects will.

2. We shall use the same draughting scale for all plans from the largest detail drawings to the smallest maps.

3. A plain metric rule, such as every man will carry in his pocket, will afford a moderately convenient scale for reading dimensions on all the commonest sizes of drawings.

4. If we write the scale as a decimal fraction there will be no ambiguity; large scales will be expressed by large numbers; small scales by small numbers.

Apply the metric system to leveling.

1. We shall have a unit of just the right magnitude for nice work. The hundredth of a foot is not considered fine enough for the most accurate leveling; so we undertake to use the thousandth: but that is too fine a division to be read twice alike. The millimeter, which is very near to a mean proportional between the two, is just what is wanted.

2. We shall have a target rod without a vernier; for millimeters can be read from a plain scale.

3. We shall have a speaking rod which can be read at a greater distance than at present, or more easily read at the same distance; for the centimeter is more than 3 hundredths of a foot. Intermediate millimeters can be estimated very closely if required.

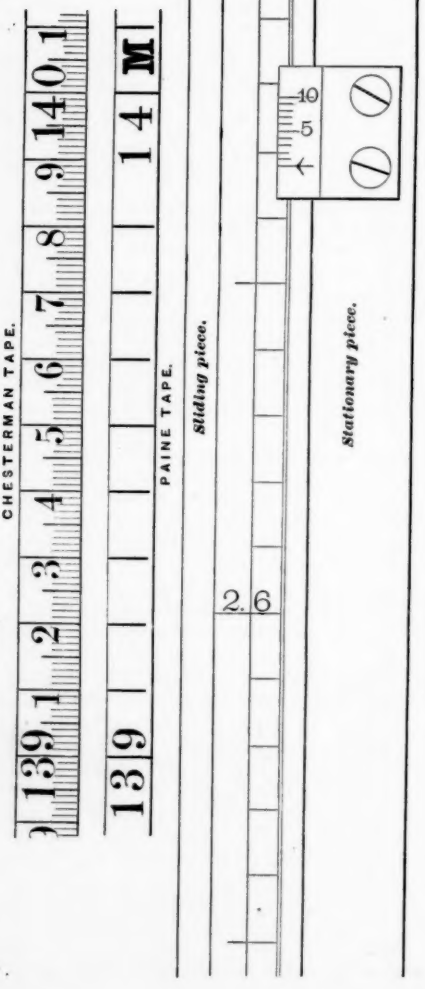
Apply the metric system to the measurement of length. As to whether it will be more convenient than the old measures, probably opinions differ according to the class of work especially contemplated by the person making the comparison. For suburban house lots where it might be thought finical to give the hundredth of a foot, and negligent to stop with the nearest tenth, the centimeter may be found better suited. For the measurement of the most valuable estates in the business portion of cities,

A SLIDE-ROD SCALE

Submitted by Charles H. Swan, C.E.
Providence R.I.

☞ *READING 2.563*

FULL SIZE.



A SPEAKING-ROD PATTERN.

Submitted by F. Brooks.

☞ *SHADED PORTION RED*

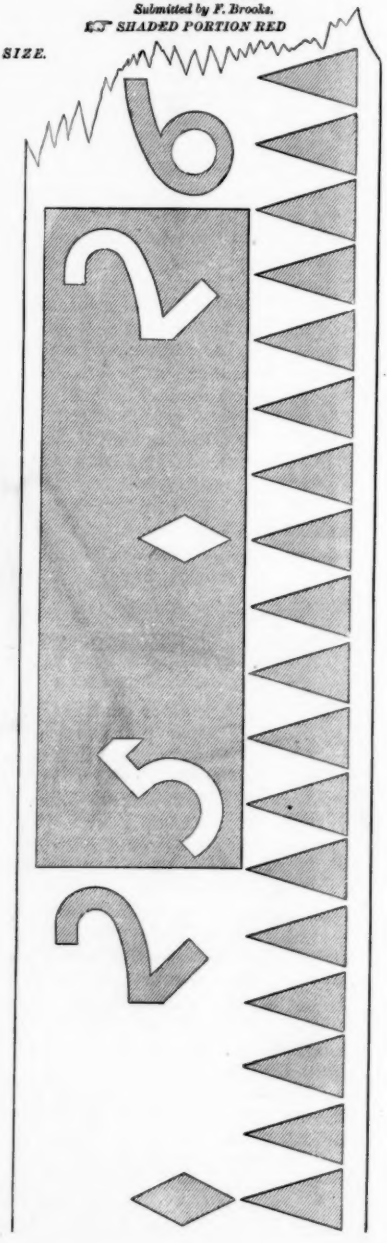
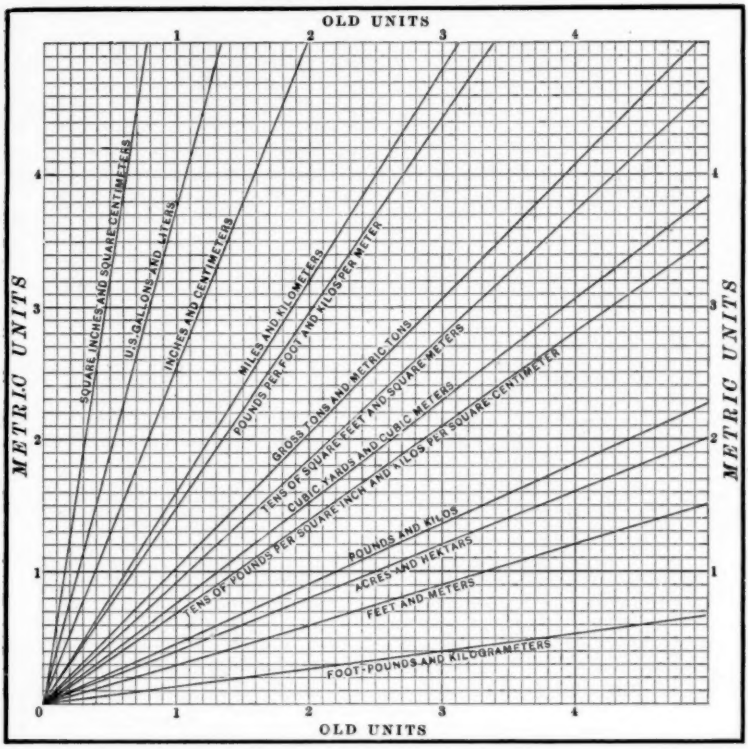


DIAGRAM OF EQUIVALENTS
(Adapted from a German periodical)


OLD UNITS
METRIC UNITS
METRIC UNITS
OLD UNITS
SQUARE INCHES AND SQUARE CENTIMETERS
U.S. GALLONS AND LITERS
INCHES AND CENTIMETERS
MILES AND KILOMETERS
POUNDS PER FOOT AND KILOS PER METER
GROSS TONS AND METRIC TONS
TENS OF SQUARE FEET AND SQUARE METERS
CUBIC YARDS AND CUBIC METERS
TENS OF POUNDS PER SQUARE INCH AND KILOS PER SQUARE CENTIMETER
POUNDS AND KILOS
ACRES AND HEKTARS
FEET AND METERS
FOOT-POUNDS AND KILOGRAMETERS

there will be some gain in nicety by the use of the millimeter instead of the hundredth of the foot, the thousandth being as unattainable here as in leveling.

We have been speaking of beginning new works of pure land-surveying. There is a great deal of surveying, however, which is more closely bound up with old measures. Where a large tract of land like our Back Bay territory has already been planned and partially laid out, it will plainly be easier to continue the use of the same unit of measurement first employed than to adopt a new one in going on to complete the work or set off additional lots. When surveys of land are made with a view to the construction of buildings or works thereupon, it would obviously be most convenient to apply the same standard of length on the survey that is to be used in the construction; in such surveys the engineer must change very nearly in unison with the manufacturers who will supply his materials in standard sizes and the mechanics who will execute his plans. Upon purely constructive works independent of surveying, the engineer must evidently act precisely simultaneously with artisans and tradespeople, in undertaking to introduce thoroughly and exclusively the use of the metric system.

Some eminent and able men have maintained that in land surveying there is need of especial delay, and that the lapse of time required for making the proposed change must there be greater than in other classes of measurement. To controvert such views we may refer to a letter which we recently addressed to the Congressional committee; a copy accompanies this report.

It is hoped that this subject will

receive the thoughtful attention of the society.

Respectfully submitted,

[SIGNED] FREDK. BROOKS,
" L. FREDK. RICE,
" CLEMENS HERSCHEL,
Committee.

19th March, 1879.

[COPY.]

BOSTON 11TH FEB., 1879.

To the Committee on Coinage, Weights and Measures:

The Boston Society of Civil Engineers having appointed us, the undersigned, a standing committee on the Metric System of Weights and Measures, we desire to present the following discussion of the arguments on the metric system in *land surveys* reported by officers of the Executive Departments and printed last year in Ex. Doc. 71, 45th Cong., 2d Sess.

The Commissioner of the General Land Office says (Part 2, p. 2);—

"The Gunter chain, so long used in this branch of the public service, is of the convenient length of 66 feet."

Now that equals 20.117 meters; and the usual metric chain has the convenient length of 65.62 feet, or 20 meters. The difference is $4\frac{1}{2}$ inches.

He says in explanation of the convenience of the Gunter chain as applied to the sub-division of the public lands (Part 2, p. 2):—

"The legal township of the United States land surveys is approximately a rectangular tract, with sides of six statute miles. This body of land is divided into 36 sections, with sides of 80 chains, each regular section embracing, as nearly as may be, a square mile or 640 acres."

But the law tolerates a variation of 1 chain in 80, and many sections actually vary from a square mile by more than half that amount. Less than half is sufficient to reduce the section to a square of 1600 meters on a side, as frequently remarked by western surveyors, and exhibited by the diagram of the Commissioner of Education (Part 2, p. 8). He shows the quarter sections and the usual smaller divisions; but the metric dimension is adapted to still further quartering than the old mile measurement. It appears therefore, that the same division now established could be continued; that what is now "approximately" 80 Gunter chains, also is, and might be called, approximately 80 metric chains, or 1600 meters; that what is now "as nearly as may be" 640 acres is as nearly as need be $(16\times16=)256$ hektars; that a 40 acre lot might as well be considered 16 hektars; and that the substitution of the metric system would thus require but a very simple computation from the old denominations to the new, and would *not* necessitate the "retracement of many standard and meridian lines now marked in the field," and would *not* destroy those associations which the land system now has with "the social and political life of the people."

The Commissioner of the General Land Office states (Part 2. p. 2);—

"The aggregate quantity of unsurveyed public lands in eleven partially surveyed States and Territories, and in the wholly unsurveyed Territory of Alaska, is about equal to that already surveyed."

After referring to the same fact, Maj. Powell, of the Rocky Mountain Survey, well observes (Part 2, p. 18);—

"If the metric system is finally to prevail in this country, it is desirable that these lands should be measured and conveyed in units of the new system."

The Commissioner of Patents relates the interesting fact that at some places in the Mississippi Valley the ancient French arpent still continues in use, and remarks (Part 2 p. 5);—

"It is because all real estate transactions are matters of permanent record, and permanent records are only changed with great difficulty. To change them involves translation, tedious and accurate computation, the discarding of original records, and opens the door to mistakes and fraud; and the possibilities of these are without end."

We see no occasion for changing or discarding the original records, but we do think that any new measurements ought to be made and recorded in terms of some unit recognized by the laws of the United States. In New England surveys were formerly made in chains and links, but it is now the common practice to use feet and decimals. Our old records remain, and at those rare intervals when we have to refer to them we can translate dimensions from one de-

nomination into the other with tolerable confidence in our accuracy, and with no special temptation to fraud. Changing arpents into feet would be the same sort of thing, though perhaps a little more tedious; and changing feet into meters we find presents no insuperable difficulty. As Maj. Powell very justly says with reference to the metric system (Part 2, p. 18);—

"In land measures its introduction will but slightly inconvenience the people at large, for the measurement of land is practically relegated to skilled persons, as engineers and surveyors; and the conveyancing of lands, to persons skilled in that branch of business."

If a change of unit were really impracticable, it would be true of the City of St. Louis, and not of its suburbs alone; but in the city the antiquated arpent has given place to another anomalous measure, the "front foot." Real estate is now sold in St. Louis, by the foot of street line, although the lots vary in depth. The whole territory of France also has found it possible to change from the arpent to the hektar.

Prof. Hilgard, Assistant in Charge of the U. S. Coast Survey Office, says (Part 1, p. 24);—

. . "all the most valuable real estate, such as lots and streets in cities, has been laid off in this country in even feet, generally even tens of feet, as 50, 60, 80, 100, 150, &c. What adequate motive is there to change these expressions into terms which are necessarily fractional, and in which those foreign nations whose convenience it is proposed to meet have no conceivable interest? What useful purpose is subserved by designating a building-lot 24 by 120 feet in the form of 7.315 by 36.576 meters?"

One weak point about this is that it does not adhere closely enough to the facts. Many of those city lots in exact feet are the lovely creations of the fancy. To draw an illustration from home, a large part of South Boston is laid out in rectangular blocks, several of which have a nominal length from street to street of 500 feet; the plans at the City Surveyor's office show the following series of real distances: 501.23, 501.37, 501.19, 500.76, 500.60, 500. 501.10, 501.23. The general fact is that cities are laid out and division walls built when land is cheap, and when minutely accurate measurement would cost more than it would be worth. In after years when the property becomes valuable and is carefully surveyed, it is found that the actual dimensions are often fractional. In such cases their metric equivalents will be no more irregular.

Whether foreign nations be interested or not, what we seek is our own convenience. In real estate business, as elsewhere, the useful purposes to be accomplished by the introduction of the metric system are so obvious that a very brief mention of them will be a sufficient answer to Prof. Hilgard's questions and to some other remarks of similar tenor, as that "there is nothing to compensate for the hardship and the danger that would ensue from such a change.

One great benefit will consist in uniformity with other measures. If land is surveyed at all it is probably with a view to its being subsequently used in some way more or less connected with such measurements as are adopted for the common purposes of life. If it is to be walled in or fenced around by masons and carpenters who use the metric system, if roads are to be made across it by contractors who use the metric system, trees planted at regular distances by farmers who use the metric system, houses and barns erected by architects and builders who use the metric system, or even graves dug in it by sextons who use the metric system, there will be an element of convenience in having the survey of the land in metric units to begin with.

A second gain will be that we shall effectually get rid of some awkward relations like 7.92 inches in a link, 16.5 feet in a rod, 5,280 feet in a mile, and 43,560 square feet in an acre, and substitute a completely decimal system with a logical nomenclature.

Another advantage relates to our dealings with distant places. If to-day a land-owner in St. Louis desires to borrow money upon his estate consisting of 25 "front feet," a capitalist outside the city is put to more trouble to inform himself about it than he will be when the description shall be made in a form everywhere familiar. If a western railroad company wishes to place its land

grant bonds in the German market, the business could be facilitated by describing the property in kilometers and hektars as used in Germany. If American instrument makers try to sell their goods in foreign countries, as American manufacturers in other departments are so generally doing, it will help them to have the same style of implements for land-measuring used both at home and abroad. Our engineers, by familiarizing themselves with metric dimensions upon their own surveys, will understand better when they read that a foreign work has drained a territory of so many hektars, or that a street in a foreign city is so many meters wide.

It is for just such reasons, presumably, that, as stated by Prof. Hilgard himself (Part I., p. 19),

"In the operation of the Coast Survey the meter is used, and has been employed from the first as the unit of measure."

We think this fact redounds greatly to the credit of the far-seeing officers who originated the practice.

It is for such reasons, probably, that the meter has been frequently used in the map-work of the U. S. Geological and Geographical Survey of the Territories, as mentioned by Prof. Hayden (Part 2, p. 15).

It is for such reasons, also, that upon several recent local surveys in our own Commonwealth the meter has been made the unit. The re-survey of Boston Upper Harbor, under the Harbor Commissioners, has been made in meters. The Boston and Providence Railroad Co. has set up a row of stone posts, two kilometers apart, along its line. Numerous small surveys for deeds of record, etc., in the towns of Abington, Rockland, and Hanover, Plymouth County, Mass., have been made and plotted by the metric system during the past five years.

If to accomplish the change in land units is going to require such an unconscionably long period as has been supposed, we can only urge that no time be lost in getting started.

Very respectfully,

[SIGNED] FREDK. BROOKS,
" L. FREDK. RICE,
" CLEMENS HERSCHEL,
Committee.

[COPY.]

To the Committee on Library of the American Society of Civil Engineers.

GENTLEMEN—May we ask your co-operation in an effort to bring about (as far as possible) unanimity of action in a little matter of professional practice? We observe in your Proceedings a standing request to members, "in papers hereafter presented to the Society to write, in parenthesis, weights or dimensions by the metric system in connection with those of the system in general use;" and we think that the spirit of this recommendation easily includes the addition of a metric scale to such plans as require any linear scale. Another announcement says with regard to illustrations of papers presented for publication, "Always put a lineal scale upon each drawing"; and this direction appears to be followed in a large proportion of cases. We would like to inquire whether in reproducing such illustrations in the Transactions an appropriate metric scale could not be added by the Secretary if neglected by the author. We append a list of several documents, plans, &c., having more or less of a public character, which bear duplicate scales, saying nothing of the private practice in engineers' offices. The metric scale is constructed graphically from the scale already on, the whole operation requiring only a few minutes time.—Respectfully,

[Signed] FREDK. BROOKS,
" L. FREDK. RICE,
" CLEMENS HERSCHEL,

Standing Committee on the Metric System of the Boston Society of Civil Engineers.

[Signed] CHAS. A. ASHBURNER,
Chairman Committee on the Metric System, Engineers' Club of Philadelphia.

[Signed] W. A. NORTON,
President of New Haven Engineering Society.

[Signed] C. W. KELLY,
Secretary of New Haven Engineering Society.

[Signed] HENRY FLAD,
President Engineers' Club, St. Louis, Mo.

[Signed] SAMUEL S. GREELEY,
Chairman of the Metric Committee of the Civil Engineers' Club of the N. W.

LIST (referred to above).

Annual Report of City Engineer, Boston, 20th Jan., 1877; Annual Report of City Engineer, Boston, 21 Jan. 1878; Report of Newton Water Commissioners, Nov. 1, 1877; Fred M. Hersey's competitive plan for a Park, Boston, May, 1878; Plan, profile, &c., Quebec W. W. in *Engineering News*, Chicago, 16th May, '78; Figs. 629–49 on pp. 656–7, 675 on p. 686, and 1016 on Plate XVII, Drinker's Tunneling, N. Y., 1878; Plan of South Boston Flats improvement, Mass. Harbor Commissioners, 28 Feb. 1878; Providence W. W. Sections and Details of Hope Reservoir, Providence, R. I. 1876 (exhibited at Centennial Exposition); 11 maps in Gray's Atlas of the U. S., Philadelphia, 1877; Boston W. W. Systems of Supply and Drainage Areas, City Engineer's Office, May, '78; Plan 1, showing the Mouth of the Rhone, accompanying Appendix S 12 to Report of the Chief of Engineers for 1875, Part 1. (Of this last probably the original scale was metric, and the British scale was added subsequently).

A NEW PROCESS IN METALLURGY.*

BY JOHN HOLLOWAY.

From "Nature."

LONG before human art acquired the knowledge of metal-making, prehistoric man had learned to make fire of the dry stems and branches of trees; in the charred fragments of half-burnt wood we recognize a form of carbon, the first simple elementary body produced by man from the complex natural bodies with which he was surrounded. In the knowledge of the use of fire, then, was the first dawn of art, particularly of that art which deals with the reduction of simple bodies from compound minerals. To convert metallic compounds into metallic elements is the domain of the metallurgist, and the means by which this is effected constitute the basis of metallurgic art. Carbon was thus a necessity to metallurgy—with the knowledge of fire the world emerged from the stone age. From those early times down to the present day, no fusion has been effected without using carbon, which in the form of wood, coal, or charcoal, has been the substance invariably used by the metallurgist for the production of heat, and to enable him to decompose and to smelt metal-bearing materials.

The new process, however, we are about to describe, has for its object the smelting of metalliferous substances without the employment of carbonaceous fuel. The sulphides of iron, lead, and zinc are known to be combustible substances of almost universal occurrence, and when burnt under favorable conditions give rise to a great evolution of heat. We have calculated the relative temperatures thus produced, from which it appears that the temperature at which iron pyrites (bisulphide of iron) burns in air under the conditions most favorable to the development of a high temperature is over 2,000° C., protosulphide of iron burning at about 2,225° C. Zinc sulphide, or blende, gives a temperature of 1,992° C., and galena 1,863° C.; while calculations made in a similar manner with coal, assuming it to be completely burnt, show the temperature attainable to be 2,787° C. These mineral sulphides, which are therefore natural and almost inexhaustible sources of heat and energy, can under certain circumstances be burnt more economically than their heat-giving equivalent of coal.

The best means, however, of utilizing this heat-producing property of metallic sulphides is not so apparent as would appear at first sight. Only iron pyrites is sufficiently combustible at a low temperature to burn in the open air, the mass

* A paper with full details of the process was read at the Society of Arts on February 12, 1879.

being raised to the temperature at which the oxidation takes place solely by the union of the sulphur and iron with aerial oxygen. In Spain this is carried on in vast heaps of hundreds of thousands of tons, and the operation extends over many months. The oxide of iron that remains is typical of those mineral substances which, once burned in the primeval operation of nature, gave up their stores of heat and force, and became, as it were, inert bodies.

Going back now to the combustion of carbon, it is well known that it burns at widely varying temperatures, as for example, in our bodies, in a common coal fire, or in a powerful furnace. A great deal of attention and thought have been spent upon the subject of the economy of carbonaceous fuel, and great advances have been made in this direction, yet the expenditure of coal or coke necessary, say, to melt a given quality of metal, still far exceeds the theoretical limit. The main causes of this discrepancy may be accounted for as follows:—(1) That only a fractional part of the oxygen of the air passed into the furnace acts upon the material to be burnt. (2) That the oxygen is not brought in contact with the combustible matter with sufficient rapidity to attain the necessary temperature for the operation. (3) That gases pass off hot and unburnt; these are now, however, frequently utilized.

There is one metallurgical operation in which the first two sources of loss are perfectly avoided—namely, by blowing air through molten crude iron, as in the Bessemer operation, where, by the burning of small quantities of carbon and silicon contained in the crude iron a very high temperature is attained, which is not the case in the process of puddling, where the oxidation is spread over a considerable period of time, although the same constituents are frequently burnt in similar proportions. But even in the Bessemer process the carbon is only half burned, and a large amount of heat escapes with the carbonic oxide and nitrogen. When, however, we blow thin streams of air through molten sulphide of iron lying upon a tuyère hearth, a high temperature is produced by the perfect combustion which ensues in the midst of the sulphides, and no unburnt gases excepting sulphur vapor escape from the surface of the molten mass. Hot nitrogen and sulphurous acid being the only gaseous products of the operation (excepting the small quantities of hydrogen from the aqueous vapor of the air), these may be caused to act upon iron pyrites and other mineral matter. When pyrites is thus heated, an atom of sulphur held in feeble combination is in great part expelled, and thus is obtained protosulphide of iron, with which the operation commences, and which can exist in the molten state. Sulphide of zinc thrown into this bath of molten sulphide is converted into oxide: the sulphides of copper, nickel, and silver do not burn at all so long as sulphide of iron is present, and, accordingly, if oxides, silicates, or carbonates of these latter metals are introduced into the molten sulphide of iron, the iron present will take away the oxygen with which the metals are combined and concentrate them into a regulus of sulphides. But the question then arises, How, after fractional decomposition by oxidation, we can separate the sulphides from the oxides? This is accomplished by the addition of siliceous matter introduced into the furnace with the charge of sulphides, so that in the manner explained are obtained from crude materials five principal classes of products, viz.:—(1) sulphur; (2) sublimates of volatile sulphides and oxides; (3) a slag of silicates of certain more oxidizable metals, principally iron; (4) regulus containing the nickel, copper, and silver; (5) sulphurous acid and nitrogen. Under certain circumstances a sixth class of products may be obtained consisting of the metals copper and lead. Thus, when the sulphides of iron and copper present in the bath are treated continuously with the blast of air without the addition of combustible sulphides, a point at length arrives when the whole of the iron present is oxidized, and the regulus in the bath consists of subsulphide of copper. If now a limited supply of air is introduced, the copper is reduced to the metallic state, with the evolution of sulphurous acid. Further experience in the matter may lead to the adoption of this continuation of the process. Again, sulphide of lead present in the bath may be caused to yield metallic lead by partial oxidation. The sulphurous acid can be made into sulphuric acid

in chambers or condensed to the liquid state. Thus we have in this new process a metallurgical operation, the necessary heat for the decomposition and fusion being entirely obtained by the combustion of the iron and sulphur contained in the materials operated on.

Some large experiments have been made in order to prove the more important points here enunciated. They are all to be found described in the paper upon the subject in the *Journal* of the Society of Arts, dated February 14 and 21, 1879. A brief record of some of the phenomena witnessed at the February experiments at Penistone may not be uninteresting. At seven in the morning on February 12 last a small party of gentlemen arrived at Messrs. Cammell & Co.'s Penistone Steel Works, in order to see the operation from its very commencement. Two Bessemer converters were ready for the experiments; one of these was charged at 10 A.M. with some molten protosulphide of iron (made by fusing some pyrites in a cupola), and a blast of air was driven through the tuyères. Lumps of sandstone were continuously thrown in together with cupreous pyrites. A flame of the burning vapor of sulphur expelled from the pyrites passed from the converter mouth to the chimney shaft; it was from 6 to 10 feet long, blue at the edges and greenish in the body of the flame. About noon this experiment broke down through an accident, after which the product was taken out. An experiment was then commenced by setting fire to some sulphide of iron by means of about 2 cwt. of coal thrown into the vessel to start the combustion; pyrites and sandstone were then thrown in, in lumps, which rapidly melted, this being continued until midnight (over eight hours). The molten mass in the vessel remaining perfectly liquid was, from time to time, partially poured out to make room for the addition of further similar materials. During the whole of the eight hours not an ounce of coal was used, the converter being "fed with stones," and "vomiting forth fire and brimstone," as a gentleman present graphically expressed it. In this latter experiment about eighteen tons of raw pyrites were thus treated, and over four tons of sulphur distilled and afterwards burnt. More than half a million cubic feet of sulphurous acid and nitrogen left the mouth of the converter at a high temperature, taking away with them a considerable fraction of the heat produced by the oxidation. This was very unfavorable to the success of the experiment, as will be readily understood when this great loss of heat is taken into account. With a suitable plant the heated gases would be utilized to drive off sulphur from pyrites, so as to produce the molten protosulphide required to continue the operation. Heat is not only obtained by the oxidation of the metallic sulphides, but also by the oxidation of iron protoxide to peroxide when the contents of the vessel are over-blown. In an experiment made in July last the oxidation was thus purposely continued. "As soon as the subsulphide of copper began to burn a splendid emerald green flame suddenly appeared, lasting about a minute, and all the lines except those of copper and sodium left the spectrum. During the last few minutes of the blow the mouth of the converter was dull and without flame."

Some of the products of these experiments were shown at the Society of Arts; they consisted of crystalline masses of ferrous silicate and blocks of fifty per cent. copper regulus. No sulphur was collected, it being impossible to do so with Bessemer plant, which, in actual operations, will not be used for the process. These experiments, however, enabled those present to witness, in the course of a few hours, the principal effects produced. "A remarkable spectrum was obtained from the burning sulphur vapor; viewed through a small direct vision spectroscope, many absorption bands were seen occurring at apparently regular intervals from the red to the violet. The lines of sodium, lithium and thallium were recognizable, but the majority of the lines are of (as yet) unknown origin, though they are the most important, since the changes furnish indications of the progress of the chemical changes taking place in the vessel. The lithium was, probably, derived from the sand introduced with the pyrites."

The process is peculiarly suitable—

(1) For the treatment of metalliferous substances which cannot be advantageously utilized by other processes. For the extraction of sulphur by distillation,

and simultaneously for the concentration and separation of copper, silver and nickel from such materials in the form of a metallic regulus; while lead, zinc, arsenic, &c., accrue in the sublimates.

(2) For the treatment of cupreous pyrites, large quantities of which exist in many parts of the world where fuel is scarce, and where the present mode of treatment by the cementation (wet) process involves not only the loss of vast quantities of sulphur, which is burnt to sulphurous acid, but causes the destruction of all vegetation within its influence. For example—about one million tons of pyrites, too poor in copper to pay for shipment to the United Kingdom, are annually treated in Spain by the cementation process. Such ores thus treated, containing 1½ per cent. of copper, leave only a small profit, whereas it is calculated that similar ores by this new process will yield a profit more than five times as great.

(3) For the treatment of copper and nickel ores, so as to produce a concentrated regulus without employing carbonaceous fuel.

It is therefore obvious that this process will effect a great revolution in the treatment of metallic sulphides, such as iron, cupreous and nickeliferous pyrites, also copper and nickel ores and the refuse gangue of mining operations, which can thus be smelted without the employment of carbonaceous fuel, the necessary heat being obtained by the oxidation of the metallic sulphides.

THE DESIGN AND CONSTRUCTION OF SEWERS.*

By GRAHAM SMITH, C. E.

From "Iron."

The expeditious removal of fœcal matter and refuse of all kinds from the habitations of the members of any community largely conduces to their individual comfort, and the health of the whole community is much dependent on the efficacy of the measures adopted to effect this end. The water-carriage system may be said to be now universally in operation in this country, and the success or non-success of this system mainly depends upon the design and construction of the sewers. These should not only be constructed in a manner such that they will carry to the outfall with despatch the sewage which may find its way into them, but they should likewise be built in a manner that no portion of such sewage should percolate through them into the surrounding earth. All matter discharged from the stomach and bowels of a cholera patient is infective, and if a taint of the infective material gains access to any well or other source of water supply, it may spread the disease indefinitely. Notwithstanding this fact, the records of the General Board of Health demonstrate that at the present time there are many branch sewers and drains existing in London which allow of the liquid portions of the sewage passing through the joints in the brickwork. These contaminate the surrounding earth, whilst the solid matters remain behind to choke the sewers and pollute the atmosphere with the gases which they generate whilst undergoing the process of decomposition. It is painfully apparent that the early sewers were, as a rule, very imperfectly constructed, and even now many sewage works are improperly carried out, simply because Local Boards are prone to practice a false economy in their public works department, and retain the moneys which ought to be paid as fees to competent men for supervising their work. An examination of Appendix No. 1 to the Annual Medical Report to the Local Government Board will convince the most sceptical that disease is caused in our country to a very large extent through defective sewers and defective drainage arrangements. Forty-two places were inspected, and, with the exception of one being a question of hospital

* Paper read at a meeting of the Liverpool Engineering Society, February 25th, 1879.

accommodation, and another to certain manufacturing processes causing a nuisance, the grounds for inquiry were the presence of zymotic diseases which were attributed by the medical officers in almost every instance to such causes as insufficient and polluted water-supply; imperfect means for the disposal of excrement and refuse; defective sewerage and drainage; and improperly constructed sewers. Sufficient has now been said to show the importance of the subject under consideration, and the effect it has on the well-being and welfare of the inhabitants of our villages, towns and cities.

The laws on which sewers should be designed, laid out and constructed, although by no means of a rule-of-thumb description, are yet sufficiently well known and understood to enable a competent man to treat effectively undertakings of this nature. When dealing with a growing town or village the conditions are complicated, and drainage arrangements may require alteration from time to time; but with thickly inhabited towns no excuse is admissible for a mistake in the form or capacity of sewers. No works or working plans should be undertaken until a thorough study has been made of all the local circumstances which are likely to affect the scheme when completed. The amount of surface drainage due to rainfall and storms, and the quantity of sewage which will require to be carried to the outfall by each vein of the system must be ascertained as nearly as possible. The position of the outfall, so as to allow of a sufficient inclination being given to the sewers of the whole system, is a matter of first importance, and deserving of the most careful consideration. The form of sewer which should be employed in any particular instance depends in a great measure on the quantity of sewage which is likely to flow through it. If a large and unvarying quantity can be depended upon, the circular section is undoubtedly that which should be employed, or even when the minimum flow is such that the sewer runs half full, the circular form has much to recommend it. Circular sewers are more easily and economically constructed than egg-shaped sewers, and are better calculated to resist the external earth-pressures. Nevertheless, where the quantity of sewage varies considerably, the egg shape should be adopted, as the smallness of the invert increases the scouring action of a small quantity of sewage, whilst the increased size of the upper portion provides for any augmentation in flow. This is so, notwithstanding that the circular sewer when running half full exposes a smaller wetted perimeter than the egg-shaped sewer with the same sectional area of stream. In all calculations with rivers and streams, it is usual to suppose that the velocity of flow diminishes with any increase of the wetted perimeter. This is not the case, however, when dealing with narrow channels, in which the increase of depth raises the velocity to a greater extent than it is diminished by the increase of friction. This was demonstrated by the experiments carried out by Mr. John Phillips with the Westminster sewers upwards of thirty years back. These experiments led him, about the year 1847, to design the egg-shaped sewer, with the major diameter one and a half times the minor diameter. A semicircular crown, struck with a radius of half the minor diameter in length. An invert struck with a radius one quarter the minor diameter, and sides with radii of one and a half times the minor diameter. These proportions are found at the present time to give satisfactory results, and have been very generally adopted wherever the egg-shaped sewer has been employed. More recently, however, he has recommended for branch sewers, in which the flow is at times small, a modification of this form in which the radius of the invert is reduced to one-eighth of the minor diameter. He likewise demonstrated by his experiments that, with a view of rendering a sewer self-cleansing, that those of moderately large dimensions should be given a rate of inclination sufficient to cause a velocity of flow of $2\frac{1}{2}$ feet per second. In Latham's "Sanitary Engineering" will be found a table calculated on this basis, which will enable an engineer at once to determine the inclination to be given to any sewer, whatever its form or dimensions, in order to produce this velocity. In small sewers a velocity of flow of not less than $3\frac{1}{2}$ feet per second should be maintained. If the sections and inclinations are properly arranged so as to maintain these veloci-

ties under all circumstances the sewers will require little or no scavenging, and, in fact, may be considered as entirely self-cleansing. In every system where the quantity of sewage is small greater inclinations are necessary than where the quantity is large, and so the lower portions of any system usually require a less inclination than the higher portions, the sewage accumulating as the outfall is approached.

It must be borne in mind, however, that although much is due to sectional form and inclination, not a little of the success of any system of sewers depends upon the manner in which the details are arranged. Especial care must be bestowed upon the form and construction of the junctions of sewers. These must always be so designed that branch sewers may discharge into main sewers, and main sewers into larger main sewers, with a velocity of flow approximating to that in the larger sewer and in the same direction as the flow already existing therein. If this point is not attended to eddies are caused which lead to deposits being formed. The level of the bottom of the smaller sewer should never be made on a level with that of the larger sewer, as the latter is constructed to carry, as a rule, a greater depth of sewage. It stands to reason, therefore, that if the bottoms of the sewers are made on the same level there will frequently be dead water in the smaller sewer, which will lead to a deposit being formed at its mouth. Looking at the matter from a purely theoretical point of view, it will be seen that the sewers should be placed relatively to each other so that the surface of the sewage in each, at the point where the sewers join, may be at the same level. It is scarcely necessary to point out that the fluctuation in the flows in sewers are so frequent that this perfection of arrangement is rarely if ever attainable in practice. It is, nevertheless, an ideal state of things which it should be the aim of all to approach as nearly as possible in actual work. The question of ventilation must also be dealt with, for the proper and efficient ventilation of sewers is one of the most important matters to be considered in any system of drainage. The temperature of the air in sewers is generally higher than that of the external atmosphere. Mr. Haywood found by his investigations, carried out during the year 1858, that the temperatures in the London sewers during the winter were 11.61° Fah. higher, and in the summer 3.12° Fah. lower than the temperature of the external air. The average temperature of the latter for the whole year was 50.24° Fah., and for the former 55.35° Fah. This fact of the temperature in the sewers being greater than that of the external atmosphere has an important bearing on the question. The higher the temperature the more quickly will fermentation and putrefaction take place, and the more freely will the gases generated in the sewers ascend to the upper portions of the system, and find their way into dwellings if proper means are not taken for their disposal and dispersion elsewhere. In a work entitled "Sanitary Drainage of Houses and Towns," by Colonel Waring, an American author, it is stated that the excrement of a typhoid patient continually agitated in contact with fresh air and a fair admixture of water passes through a series of complete chemical changes with no injurious product, but if allowed to remain stagnant, if not properly exposed to the air, or if it gain access to the human circulation before a certain oxidation, it will, like a ferment, reproduce itself and give rise to the conditions under which it was at first produced." . . . "Unventilated and badly constructed sewers are sure agents for the propagation of the disease when once it has taken root." The latter fact is uncontrovertible. Many means have been suggested for ventilating sewers by shafts, fires, and other of the ordinary expedients for producing air-currents. These have all more or less failed, their effect having been found to be entirely local. It is now the generally accepted opinion that the only means of ventilating sewers is by allowing them to ventilate themselves. All manholes should therefore be provided with iron gratings or with ventilating pipes, so that they may not only serve the purpose for which their name denotes they are intended, but at the same time allow of the free entrance and exit of the air to the sewers. By this means the poisonous nature of the gases is to a great extent counteracted, as when largely diluted with air they are considered to be comparatively harmless.

Manholes should be placed at the junctions and angles of all main sewers, and for purposes of ventilation the distance apart of either manholes or ventilating shafts should not exceed 300 feet. No rules can be laid down for the spacing of manholes, but the more frequently they occur the better will be the ventilation and the greater the control over the system. If all gulleys and catchpits are efficiently trapped, and the junctions of all branches made in such a manner that they are never water-locked by the sewage in the main sewer, these manholes and ventilating shafts may be made to act as down-air shafts, and pipes three or four inches in diameter carried from each private drain up the side of the house, which such drain serves, to a few feet above the top of the house, will form efficient up-air shafts. The drains should be trapped just outside the walls of the dwelling. These pipes will then not only serve the purpose of up-air shafts from the main system of sewers, but they will likewise relieve the trap from all pressure and prevent the possibility of any leakage of sewer-gas into the house. This, of course, supposes that no other drains pass under the houses unless bedded in concrete, and trapped at each end immediately outside the walls of the building. Time will not permit the subject of manholes, traps and gulleys, being gone into more fully, the author will therefore devote his remaining remarks to the materials employed in the construction of sewers.

Lime mortars were often necessarily employed before the introduction of Portland cement, and even at the present time chalk and other rich limes are not unknown in sewer construction. These are not capable of resisting the chemical action of sewage, and therefore, although the more hydraulic limes may be capable of affording a sufficiently strong cementing material, they should never be employed, excepting on the crowns of sewers, which are estimated, under normal circumstances, to run about half bore, and even under these conditions, their use in sewer construction is attended with some trouble and risk. For the purpose of backing up a sewer, or filling up a trench in land liable to a settlement, good hydraulic lime concrete is all that can be desired. The water used in mixing the mortar or cement should be free from all organic matter, as should also the sand and gravel which are to enter into the construction in any form whatever. In districts where gravel, or other suitable materials for the making of concrete are to be procured, no better material than Portland cement concrete is to be found with which to construct an efficient and economical sewer. The proportions adopted by engineers in mixing Portland cement concrete for sewer work vary, but 5 to 1 and 7 to 1 may be taken as the limits employed for the main portions of the work. In some cases the whole interior of a sewer is rendered with a thin layer of pure cement or cement mortar, consisting of equal parts of cement and sand. A somewhat more easily-constructed sewer than that in which concrete only is employed is the sewer with an internal ring of brickwork, surrounded entirely with concrete. It is slightly more expensive, but it permits of the invert being formed with hard bricks glazed on one side, such as the Staffordshire blue bricks. The inverts of sewers should always be formed either with these hard bricks or with glazed earthenware blocks, as the friction and erosive action of the detritus carried in the sewage is very great. For sewers of moderate size the earthenware blocks have many advantages, as, when laid on a bed of concrete the bottom of the sewer can be constructed with facility and accuracy. The invert block in one piece is preferable to that made in three pieces, as it is difficult to find men who will make the joint of the latter properly, and when one block only is used it is generally made hollow, and so a continuous subsoil drain is formed under the sewer to carry off water from the trenches during the progress of the work. In large sewers the inverts are preferably made with bricks of the hard and smooth description already pointed out. Softer bricks, such as the "London stock," are, however, more suitable for the construction of the other portions of the work, owing to their offering a better surface for the adhesion of the mortar. A few experiments, carried out by the author with a view to ascertaining to what extent the nature of the brick affects the strength of brickwork may not be here

out of place. In the first experiment twelve bricks similar, but slightly harder than London stocks, were crossed in pairs and united with Halkin lime mortar, mixed in the proportion 1 of lime to 2 of sand. At the end of 168 days the six pairs of bricks were drawn asunder with an average of 553 lbs. for each pair. With hard firebricks tested in every way in a similar manner, the average of six experiments was 474 lbs. for each pair. These experiments tend to show that soft porous bricks are preferable to hard vitrified bricks where tensile strains have to be met in brickwork. For sewers of small diameter glazed earthenware pipes of circular section are preferable to any other system of construction. These pipes should be laid on concrete, and where the roots of trees are numerous they should be entirely surrounded with this material. Puddled clay was at one time much employed for the making of the joints of such pipes, but it has now been superseded by Portland cement, as it was not found to answer satisfactorily. The roots of trees find their way into the sewers through the small fissures which are always liable to be formed in puddled clay, simply by the dryness of the surrounding earth. In conclusion, the author deems it well to remark that the time at his disposal for the preparation of this paper having been limited, he has not been in a position to prepare diagrams, and therefore has been unable to deal in detail with the design and construction of sewers.

THE PUBLIC WORKS OF THE UNITED STATES.*

By M. MALEZIEUX, Ingénieur en Chef des Ponts et Chaussées.

Translated by DESMOND FITZGERALD, Superintendent, Western Division, Boston Water Works, for VAN NOSTRAND'S MAGAZINE.

THE NAVIGABLE WAYS OF THE UNITED STATES.

I. GENERAL OUTLINE OF EXISTING ROUTES.

Every one knows that the basin of the Mississippi, lying between the Alleghany and the Rocky Mountains, embraces about half of the United States; that the Eastern slope of the Alleghanies presents but a narrow space along the Atlantic in which colonization was nearly confined in the last century, and that on the opposite side, west of the Rocky Mountains, the continent forms an undulating plateau which has a width of nearly 2000 kilometers and more than 2000 meters average height.

Until 1847, when gold was discovered in California, a few pioneers only had ventured beyond the Mississippi and Missouri, into the plain which rises by gentle slope for 800 kilometers in distance to the foot of the Rocky Mountains. To-day one may say the center of gravity of the American continent is found on the Mississippi between St. Louis and Chicago.

It is here the great grain-producing states are grouped. The fundamental problem connected with the establishment of routes of communication, stated in a word, consisted in uniting Chicago with the Atlantic coast from which all the unconsumed products are exported. With this problem the three great navigable routes of the United States are connected.

1. The line from Chicago to Montreal by way of the Lakes and the St. Lawrence.

2. The line from the Eastern extremity of Lake Erie to New York, via. the Erie Canal and the Hudson River.

3. Lastly, the Mississippi from whose mouth vessels sail by sea for the large ports of the Atlantic, Baltimore, Philadelphia, New York, Boston.

I will now enlarge on some of the details of these lines to which will be added some remarks on the most interesting affluent of the Mississippi, the Ohio.

* In an introduction to this article, which is omitted, the author speaks of the collection of drawings and models sent from the United States to the Paris Exposition of 1878, and of the imperfect idea that they conveyed of our great public works. He was asked to describe them more fully and this paper is the result.—D.F.

THE CANADIAN LINE.

Every one knows that the waters of Lake Erie empty into Lake Ontario by way of the Niagara river. A long while ago, these lakes were united by a navigable canal—the Welland Canal. It was constructed for boats of 500 tons. It is about to be enlarged to give passage to vessels of 1000 tons which navigate the lakes.

Six canals, varying in length from 1,200 meters to 18 kilometers were constructed a long time ago around the St. Lawrence Rapids above Montreal. These canals serve for all ascending boats and in descending for all boats loaded with cereals. Rafts and steamboats descend by the river itself. As with the Welland Canal, there is a project for their enlargement. These canals were all built by the Canadian Government. To this end it has expended the sum of about 100 millions of francs. It is estimated that 90 per cent. of the merchandize transported by this route comes from the United States.

Below Montreal attention is being paid to the improvement of the river, to render it navigable at all seasons for English packets.

THE ERIE CANAL.

The Erie Canal, completed in 1826, has given to New York for more than a quarter of a century, a marked superiority over all other ports of the Atlantic. But this state of things has been modified within the last dozen years, especially by the railways. These new routes (it is said) carried in 1876, 83 per cent. of Western freights, while the Erie Canal carried only 17 per cent.

The canal was at first constructed with a depth of 1^m 22 for boats carrying 76 tons. It was soon enlarged by doubling the locks, increasing the draft to 2^m 13 and the tonnage of the boats to 240 tons.

This enlargement reduced the price of transportation 50 per cent., in conformity with the predictions of the engineers.

THE MISSISSIPPI.

Before the establishment of railways, New Orleans (thanks to the Mississippi) was the New York of the South.

This river is navigable as far as St. Paul, nearly 4000 kilometers.

But from St. Louis to St. Paul navigation is precarious on account of shoals and ice. The depth is reduced sometimes to 1 meter. In 1870 the improvement of all the rapids which are found in this portion of the river was accomplished, viz: at Keokuk and at Rock Island.

At Keokuk a lateral canal was constructed. At Rock Island a channel 60 meters wide 1^m 22 below low water was excavated in the rocky bed of the river.

From St. Louis to New Orleans, over a distance of nearly 2000 kilometers, navigation is but rarely impeded by low water or ice. The average minimum depth of water has varied during the nine years from 1868 to 1876 as indicated in the following table:

	Days.
Less than 1^m22 during	5
More than 1^m22 and less than 1^m83	52
More than 1^m83 and less than 2^m44	103
More than 2^m44 and less than 3^m05	69
More than 3^m05	136
	365

Vessels coming from the sea rarely pass New Orleans, but whatever their draft, they can always ascend as far as Vicksburg and sometimes to Memphis, (1,200 kilometers from New Orleans). From New Orleans to the apex of the delta, for a length of 195 kilometers, the depth of the water is about 30 meters.

All the different passes into which the river divides itself have a bar at their extremities in which the natural depth is less than 5 meters.

As long as commercial navigation was limited to vessels of 400 to 500 tons, drawing from 3^m to 4^m25 of water, the bars at the mouth of the Mississippi presented no serious obstacle. But for about the last quarter of a century, maritime commerce has employed, commonly, vessels of 1000 to 5000 tons, with a draft of from 16 to 23 feet (from 5 to 7 meters). Experience has proved their economy beyond a doubt, notably, for long voyages. Since that time New Orleans, (like Nantes) has scarcely been more than a port for coasters.

For several years preceding 1875, the Federal Government has expended in the neigborhood of 1 million francs in annual dredging, in the South West Pass. A channel from 15 to 20 feet deep (4^m57 to 6^m10) and from 15 to 60 meters wide was thus obtained. But the result was in-

sufficient and doubly precarious, storms sometimes filling up to to-day the channel of yesterday, in spite of appropriations.

THE OHIO RIVER.

The Ohio drains in the upper parts of its course carboniferous regions whose area is larger than those of Great Britain, France and Belgium united.

Pittsburg is the center of the coal commerce of the West and South West. In 1876 above 2,500,000 tons of coal were shipped on the Ohio from this point.

From Pittsburg to Cairo, where the Ohio joins the Mississippi, is about 1550 kilometers. Though much less troubled with ice than the Upper Mississippi and the Missouri, the Ohio is subject to greater freshets than any of the other affluents of the great river. The water rises 19 meters at Cincinnati. The coal barges require a depth of 1^m83. Above Cincinnati the depth is sometimes as little as 2 to 3 feet. Below, (that is to say for a distance of 829 kilometers) steamers large or small can generally navigate during eleven months in the year.

The canal from Louisville to Portland, (constructed to avoid the rapids) has been wholly in the hands of the Government since 1874. It cost $16\frac{1}{2}$ millions of francs. By reducing the toll from 2 fr. 50 to 0 fr. 50 per ton a considerable service was rendered to the navigation of the Ohio.

The freight on coal from Pittsburgh to Louisville or to New Orleans is from one to two millimes (0.02 to 0.04 cent.) per ton kilometer. There is not a railway in the world that could carry at so low a price and, consequently, the Ohio is the only route which can supply coal to the centers of populations established on the banks of the Mississippi below St. Louis; to the steamboats which ply its waters and to those which take the sea at its mouth.

PROJECTED CONNECTIONS.

Among the number of connections that are projected between certain navigable ways may be mentioned the following:

1. Canal of large section from Lake Champlain to the St. Lawrence (Caughnawaga).

2. From Troy to Oswego (lake Ontario) by way of lake Oneida.

3. From the James River to the Kanawha (or from Richmond to the Ohio).

4. From the Tennessee to the Chattahoochee (by Macon).

5. From Rock Island to Hennepin.

II. THE EXECUTION OF THE WORKS.

Rivers can be improved with a view to navigation, by dredging, by lowering their bed, by widening, by ridding them of rocks. The end sought is thus attained directly. It can also be attained indirectly by narrowing the bed by longitudinal dikes which concentrate the flow.

Lastly, an improvement much more radical can be obtained by controlling by dams the course of the river, diminishing the velocity and at the same time increasing the depth, in a word, transforming the river into a series of lifts like those of a canal.

A. DAMS.

Americans have no movable dams although they have some fixed ones, and also some canalized rivers. But this question is the order of the day with them—they come to study what we are doing in France, desirous of ascertaining how our various systems can be applied, notably to the Ohio—and to water courses much larger than ours, where the floods are much higher and more sudden and where very few men can be had for manœuvers.

Under these more complex and difficult conditions, American engineers are now studying the problem of movable dams. No doubt but that their spirit of invention will be given full play, and that some day at some new international exposition they will return us with interest the lessons that France is happy enough to furnish them to-day.

Leaving the subject of dams, there only remains for us to examine in America dredging machines and examples of channels narrowed by dikes.

B. DREDGING MACHINES.

Spoon Dredge—Although chains of cup dredges are found in the United States, the apparatus most commonly employed in rivers and canals is a dipper dredge directed by an arm.

It has been sought from time to time in Europe to solve the problem of the managament of this arm, which is 10 or

12 meters long. It is held by and turns on a roller, whose axle carries a wheel joined by a chain to the two extremities of the arm. One man governs it easily. This machine, for which perhaps Americans have too exclusive a preference would be useful in France for accurate dredging.

Shell Dredge.—The shell dredge is an apparatus of which the original idea belongs to Europe, but of which Americans alone have been able to make a practical, accurate, strong and useful machine. The difficult point was to produce from above, by means of a chain, a downward pressure, causing two shells or jaws, mounted on a common horizontal hinge, to penetrate the soil and to join after imprisoning between them the detached materials.

Rakes are employed in the Upper Mississippi to increase, at a small cost, the depth of the channel from 1 meter to 1^{m}30 on certain sandy shoals. They have been found so successful that the ordinary boats used in commerce sometimes ask for these rakes to attach behind.

The General MacAlester dredge-boat is represented at the palace of the Champ du Mars, as are also the preceding machines.*

Removal of Sub Marine Rocks.—A new method for removing large masses of sub-marine rocks was applied at San Francisco in 1870, and since then has been used at New York as follows:

First a coffer dam is established on the summit of the rock from the interior of which, after being emptied, a well is sunk to the level of the projected excavation.

Galleries are opened radiating from this well, joined by concentric galleries or else by galleries cutting each other at right angles with proper piers, and a roof of sufficient thickness to exclude the water. Thus the whole mass is excavated. So well was this done at San Francisco that at the last, wooden posts were substituted for the pillars which had been left. Finally, casks of powder are distributed in the interior, the water is let in, and then, at high tide, a simultaneous and general explosion takes place by means of electricity. In this system a grade is fixed upon a little lower than the level required for the channel, in order to dispense with picking up the fragments of debris from the excavation, which is sometimes expensive.

Among the processes of sub-marine blasting may be always found that which consists of sinking holes, in which cartridges are placed. For this purpose steam drills mounted on flying scaffolds are used.

In a model (not on exhibition) which was shown me several weeks ago by Mr. Julius H. Stredinger, (one of the assistant engineers to Gen. Newton, at Hell Gate, near New York) the scaffold was not fixed, but was held in place by four sliding legs which carried it, and which is made to rise or fall by means of racks. When it is wished to move the apparatus, a special ponton is advanced and inserted between the legs, and these being raised the pontons support the scaffold which can then be carried somewhere else.

This method, already employed in spoon and shell dredges, avoids the use of long guys which are often so troublesome to other boats or vessels.

C. DIKES FOR NARROWING THE CHANNEL; THEIR USE FOR IMPROVING MOUTHS OF RIVERS.

1ST. TRIBUTARIES OF THE GREAT NORTHERN LAKES.

Americans have improved several mouths of rivers by means of dikes. Combined with dredging, this system has succeeded in opening ports at the mouths of streams which flow into the grand lakes of the North. The bars which obstruct these streams are principally produced, as at the embouchure of the rivers of the Baltic, by the action of the waves on the movable material of the shore.

It can be seen, consequently, that by means of parallel jetties prolonged a sufficient distance from the shore, it has been possible, either to prevent in the future the changes of the shore, or to arrest for a longer or shorter period the movable matter which is carried along the coast, as has been done since 1824 at the mouth of the Oder.

* This machine perfected by Major Howell, was lost in the sea last winter, while *en route* to the mouth of the Sabine River.

Below will be found a table of depths obtained on the bars.

	Originally.	In 1875
	Meters	Meters.
At Chicago.....	0.90	4.57
At Milwaukee.....	2.13	5.18
At Racine.....	0.60	4.27
At Michigan City..	0.30	3.66
At Erie.....	0.90	4.57
At Buffalo.....	0.20	4.57

2D. THE MISSISSIPPI.

A much more important undertaking has been in process of execution since 1875, viz., the improvement of the embouchure of the Mississippi.

It was debated at first whether it was best to construct a lateral canal, that is to say, a canal connecting the deep portion of the river with the sea, or to undertake the destruction of the bar

The Government engineers, in 1874, submitted a plan for a canal as the only certain solution of the problem. This canal, 11 kilometers long, was to be connected with the river near Fort St. Philips—60 kilometers from the mouth—and from this point was to take an easterly direction in a straight line and empty to the South of Breton Island. The other plan has also found strong supporters, who have protested against the idea of subjecting such an important commercial undertaking to the disadvantages of a lock, and dwelt on the benefits of a navigable pass of sufficient capacity and practicable at all times.

In a word everything possible has been said and written on the subject exactly as was done in France, when, in reference to the embouchure of the Rhine, the same question was submitted to a commission instituted by ministerial decree of Dec. 14, 1843, and whose inquiries were started on a similar plan by Engineer Turrell.

In this situation the President of the United States also appointed a commission who began by visiting all similar works in Europe, and who selected as was done in France in 1852 the plan of improving the bed of the river.

Unlike the work done for the Rhone which was considered as an experiment and for which the small sum of 1,500,000 francs was expended, the dikes designed by the American Commission were estimated to cost a much larger amount of money.

But the partizans of the system, at whose head was Mr. James B. Eads (of St. Louis, Mo.,) offered to execute the work at their own risk. This offer was accepted by Congress, March 3, 1875 and in June following Mr. Eads began work.

Let us examine the question from its technical point of view: What were the arguments entered into on both sides; what works have been executed, and what have been the results up to the present time?

Description.—The head of the passes is 20 kilometers from the sea. The river here divides into three branches which are continually extending, while the bars at the mouths, caused by the deposits of mud, advance into the sea.

The river transports annually in the neighborhood of 190 millions of cubic meters, or more than ten times that carried by the great Rhone.

The South Pass, which was chosen for improvement, has hardly 700 feet (213 meters) of average width. The width of the bar, measured on the axis of the channel between the two curves of $6^{m}.70$ depth, was 3,600 meters in June, I875, for about 800 meters the depth of the water was scarcely more than $2^{m}50$, consequently his South Pass was seldom or never used by Commerce.

I will add that from 1838 to 1874, during a period of 35 years, the bar had advanced about 30 meters on an average per year (while the annual advance of the bar in the South West Pass had been 90 meters).

Contract.—Mr. Eads was authorized to execute such jetties as he might think proper to produce a wide and deep channel across the bar at the mouth of the South Pass.

The width to be secured between the jetties was not to be less than 700 feet.

The contractor is engaged to secure the following results:

1°. In 30 months from March 3, 1875, a depth of 20 ft. ($6^{m}10$) at least, this depth to be reckoned from the lowest level of high water in the sea where the stream is at low water.

2° A farther deepening of at least 2 feet ($0^{m}61$) in the course of each of the three succeeding years. The whole

depth thus becoming 26 ft. (7^m93). If either of these conditions are not fulfilled the contract becomes null and void.

On the other hand, Mr. Eads guarantees to obtain 30 feet of water (9^m15), in consideration of which the Government is pledged to pay him, first, the sum of $5,250,000 (26,250,000f.) for the jetties and other works which become the property of the Government; second, an annual sum of $100,000—for the maintenance of the works as soon as the depth of 20 ft. is obtained—up to the final delivery of the works.

Payments of $500,000 on account, will be made when the minimum depth reaches successively 20, 22, and 24 ft. on minimum widths respectively of 200, 250, 300 and 350 ft. The Government will retain as a guarantee the last million dollars for 10 years and $500,000 for 10 years more.

In addition to which the delay of the guarantee will cease to run, and the annual subsidy of 100,000 francs will cease to be paid, during periods when the channel shall be less than 30 feet deep or 350 feet (106^m75) wide.

Lastly, the government will have authority to take possession of the jetties at any time, by paying the contractor the sum retained as guaranteed and releasing him from all responsibility.

The Act of Congress of March 3, 1875, explicitly provides that Mr. Eads shall have full liberty as far as relates to the line of the jetties, their construction and method of execution; but stipulates that they shall be substantial and durable and capable of indefinite maintenance at a reasonable annual expenditure.

In any event the Government will not be responsible for losses that Mr. Eads may sustain in the course of the work.

Principle of the System.—Before mentioning what tnis bold contractor has done and obtained up to this time, it is well to indicate the basis of his confidence in the success of the plans.

The South Pass of the Mississippi presents the characteristic features of rivers flowing with tideless seas. After remaining constant for 15 kilometers the transverse section changes.

The banks created by the river itself widen as they approach the sea in which they disappear, at the same time the latter is raised by a slope the back bone of which, constituting the bar, is found some kilometers distant. By prolonging the banks artificially by two jetties it is evident that the bar will be attacked. It will re-form a little farther out as was the case with the Rhone, but a temporary deepening will result. Here is the point made by Mr. Eads. If, said he, jetties sufficiently long are built rapidly in a few years, so that the deposits will take place in the deep portions of the sea (where the gradual foundation on which the bar will elevate itself will take place) the re formation of the obstacle can be postponed for a century. It is sufficient to take suitable advance on the natural elongation of the pass; for an elongation of 30 meters a year 1 kilometer of dike will give more than 30 years of advance.

EXECUTION.

By the last of the year 1876, the South Pass terminated in an artificial channel 300 meters wide, comprised between two parallel dikes constructed at the same time over their whole length up to a point beyond the crest of the bar where there was originally 35 feet of water ($10^m. 67$).

These dikes are similar to those of Holland, gravel and stone being lacking at the mouth of the Mississippi; one of these was 3650 meters long, the other 2500.

The scour has been further aided by numerous transverse dikes and by dredging.

If one imagines the channel divided into six portions of 600 meters each the deepening obtained from June 1875 to November, 1876 has been:

$^m.50$ in the 2d portion.
$1^m.33$ in the 3d portion.
$3^m.35$ in the 4th portion.
$3^m.47$ in the 5th portion.
$3^m.39$ in the 6th portion.

On November 14, 1876, the two 20 ft. ($6^m.10$) curves on each side of the bar were joined (reckoned from the average sea level at high tide and at low water of the river).

The minimum depth of $6^m.10$ having extended over a width of at least 60 meters, Mr. Eads received his first payment, on account, of 2,500,000 francs, in the Month of January, 1877. December 15, 1877 the depth having reached $6^m.70$

over a width of from 60 to 80 meters Mr. Eads received his second payment of 2,500,000 francs.

Present Situation.—Since then six months have passed. What is the present condition of things? Is Mr. Eads still confident of success? I have before me the three following documents:

1. A petition signed the 7th of May 1878, from Mr. Eads to the Secretary of War, with a view of obtaining several modifications in the conditions of the contract.

2. A report on this subject from a special commission consisting of two Government engineers.

3. Lastly, a letter from the Secretary of War transmitting the document to the Senate approving the conclusion of the Commission.

Mr. Eads states that the expenditure made to accomplish the results have considerably exceeded his anticipations, and that the interest is exhausting him. He asks that the time during which the successive depths should be maintained, in order to secure a new payment on account, should be shortened.

Commerce has the greatest possible interest in as speedy a termination of the works as possible. Equal profit to the public treasury would ensue, relieved as it would be from interest paid the contractor of 5 per cent. in the grants which have been complied with in principle but whose payment is postponed.

Besides this, Mr. Eads asks for a provisional reduction of the minimum widths of the channel for the successive depths; remarking that the Commission of engineers of 1874 who prepared the contract abstained from fixing any width, and that this stipulation added afterwards by a legislative committee, assumed relations between the depth and width of the channel which could not be realized in practice. Thus to obtain finally a width of 350 feet (107 meters) a greater depth of channel than 30 feet ($9^m.15$) would have to be secured, and that these dimensions exceeded the real needs of navigation. Mr. Eads concludes by asking:

1. That the progress of the deepening be verified foot by foot.

2. That the corresponding widths be modified, following a scale which he indicates. That instead of the increase from 250 to 350 feet when the depth is increased from 24 to 30 feet, the width, on the contrary, shall decrease from 150 feet to 100.

3. Lastly, that the payments be divided with 7 subsidies, diminishing concurrently from $750,000 to $250,000, the whole not to exceed the limits of the total grant.

The reserve guarantee would be maintained at one million dollars.

Lastly, Mr. Eads urges, with incontestible eloquence, the right which he has to the concessions that he solicits.

In the report of Messrs. J. G. Barnard and H. G. Wright, let us consider only some of the technical reviews of the work executed.

To attain the character of solidity and of durability which the law requires, the jetties should be enlarged and riveted with stone. This treatment is particularly necessary at the extremities of the jetties where the dikes subside from the settling of the mattresses and the subsoil, while the sea breaks over them.

The Commission of 1874 estimated that $\frac{1}{3}$ of the jetties would be made of stone and $\frac{2}{3}$ fascines. They had anticipated that more than 150,000 cubic meter of stone would be necessary. Instead of this, only enough stone has been used to aid the sinking of the mattresses, with about 15,000 cubic meters added; and Mr. Eads announces that half as much is delivered. In this case, he is far from the estimated section. However this may be, the Commission is of the opinion that the works are being properly executed, that their success is probable, and that now the government and the country are both interested, not only in the continuance of the work, but in its rapid execution. The Commission argues with the petitioner that 26 feet depth ($7^m.93$) is sufficient, and that if 30 feet was talked of it was to allow a margin.

The statement is added that on the bars of the Mississippi the whole depth of the water is available. By way of information the report adds that 85 per cent of the entire shipping of the world does not draw more than 23 feet (7 meters), that 26 feet ($7^m.93$) draft is the maximum required for the regular traffic on the bar at the entrance of New York. Lastly, that the largest vessel that ever crossed this bar drew only 28 feet ($8^m.54$).

The report of the Commission is evidently favorable as a whole to Mr. Eads' petition. It advises, however, that Congress alone can authorize the modifications asked for, as far as relates to the widths and to the payments of the subsidies.

This work so full of interest is unrepresented at the Exposition. It is nevertheless remarkable from more than one point of view.

This is one of the boldest enterprises that has ever been risked by private energy.

The results already obtained surpass from a professional point of view, and in material importance, all that has previously been done for the jetting of rivers.

If the results are obtained and completed without the jetty system proving too expensive, a veritable commercial revolution will have been accomplished not only for New Orleans but for the whole basin of the Mississippi.

§ 2.

RAILWAYS—LONG SPAN BRIDGES.

The most important works of art necessary for railways or roads are those which enable them to cross streams.

In America these are almost the only works which merit attention, for extensive cuttings are rare, large embankments rarer still, subterranean works are few and of short length, if we except the Hoosac tunnel, 7 kilometers long, constructed at the expense of the State of Massachusetts to shorten the distance from Boston to Albany and Buffalo. (This tunnel was opened for travel in 1876).

In the greater part of the United States, good building stone is lacking, thus bridges are constructed only in wood and iron. Wooden bridges, the only ones known up to about 1840, are not built now, except as a temporary measure, or when sufficient capital at first is lacking. This is the reason why almost all of the interesting bridges are of iron.

Another notable fact in the United States is that the rivers if they are few in number are, on the other hand, very large.

Their general direction is North or South, like the Alleghanies, the Rocky Mountains, or all the great physical features of the American continent. Thus they constitute barriers which averted the western expansion of colonization, first at the Ohio, then at the Mississippi and, lastly, at the Missouri. The necessity of crossing these streams arose.

Another difficulty of the problem came from the nature of the beds of these rivers which are composed in the majority of cases of moving sand, 10, 20 or 30 meters deep. The difficulties attending the foundations resulted in spacing the points of supports much farther apart than is generally the case in Europe.

Iron bridges of large span form a truly original feature in the public works of the United States. The construction of some of these bridges has cost one million dollars, others two millions, and the St. Louis bridge cost much more. Eminent engineers occupy themselves wholly with iron bridges; large companies have made an almost exclusive specialty of their construction. The Keystone and the Phoenixville of Philadelphia; the American Bridge Co. of Chicago; the Delaware Co. of New York, and the one directed by Mr. Pope in Detroit; the Watson Co. at Paterson, N. J., besides others.

TABLE.

1ST. FIXED SPANS OF TRUSS BRIDGES.

	Spans. Meters.
In 1862 there were in the United States (we believe) but 2 long span bridges, these were designed by Mr. Albert Fink; the spaces were.....	61.00
Then the Monongahela bridge was built..	79.30
And another on the Ohio at Steubenville.	97.60
In 1869 2 others on the Ohio { at Bellaire.. " Louisville	106.75 122.00
In 1871 over the Missouri at St. Charles.	91 50
In 1871 over the Ohio at Cincinnati.....	156.00
In 1871 over the Hudson at Poughkeepsie one was commenced which will have 5 spans of.....	160.12

2D. DRAW BRIDGES—WITH TWO SYMMETRICAL TRUSSES.

		Meters.
Length of the floor	{ at Chicago	68.62
	{ at Cleveland.....	99.12
Over the Mississippi	{ at Dubuque... }	109.80
	{ at Kansas City }	
	{ at Keokuk..... }	118.03
From 1873 to 1875 over the Missouri at Atchison.....		111.32

[A similar bridge is about to be erected at the port of Marseilles. The plan was

approved April 1, 1878, by the Minister of Public Works. The total length of the floor, including a bearing of $0^m.90$ on each of the two abutments, will be $73^m.80$].

3D. SUSPENSION BRIDGES.

		Meters.
1855	Niagara Bridge (lower) 2 stories	250.20
1860	Pittsburgh Bridge	105.00
—	Wheeling Bridge over the Ohio	308.05
1867	Cincinnati Bridge over the Ohio	322.00
1869	Upper Niagara Bridge	386.44
1877	Minneapolis near St. Anthony, Min.	205.90
1877	Point bridge over the Monongahela. at Pittsburg	244.00
1877	East River bridge under construction	493.00

4th. ARCH BRIDGE AT ST. LOUIS.

A central arch of	158.60
Two others of	157.07

SUPERSTRUCTURE.

A. *Pin Trusses.*—All the large bridges consisting of straight trusses continue to be built on a system much better known to-day in Europe than it was in 1870. The characteristic features may be stated as follows:

A chord of channel beams of 3 or 4 meters length, often united by cast iron boxes, and another chord of eyebars united by pins are connected by posts on ties variously combined, always jointed by pins on the lower chord and sometimes on the other in such a way that each member is never subjected to but a single stress, tension or compression, of which the maximum determines the section necessary to be given to each piece.

This method should conduce to a reduction of weight and a consequent economy. But this is not the only advantage. Others may be found in the small opportunity offered for rust, the metal following the lines of strains; in the opportunity for inspecting and repainting from time to time; in the facility of transportation—each piece isolated having but short length; in the small area offered to the wind by the large lattices, which also do not retain the snow upon the floor of the bridge.

In fact, the persistent use of the system in the United States proves conclusively that it preserves all its merits in the eyes of Americans.

Riveted lattice bridges are only employed in a limited measure. They seem to constitute a specialty for a firm established at Rochester (State of New York)—the Leighton Bridge & Iron Works.

The comparitive merits of the two systems, which differ essentially in the methods of uniting the members, is difficult to determine on account of the lack of comparative data on the weights of metal employed and the cost of labor, as well in the workshop as in erection.

There is another consideration which prevents our employing pin bridges in France.

While the iron and steel employed in American bridges is always of a superior quality, the manufacturers in Europe where riveted bridges are made, are far from regarding a rigorous choice of iron employed in their bridges as equally necessary. Then again these shops have not the special tools which allow Americans to manufacture, so surely and economically, pieces of which the types are limited. These special tools our workshops could procure, but they do not do so for the reason that they would not have use for them.

B. *Suspension Bridges,* invented in America near the end of the last century, have been perfected during the last 30 years.

By an excellent combination of suspended cables and longitudinal trusses and ties uniting the floor to the supports of the bridge; by the inclination given in plan to the cables and the addition of exterior guys; lastly, by improvements introduced in the manufacture of and methods of attaching the wire cables, Americans have succeeded in building bridges, more expensive without doubt, but perfectly substantial, and which have solved problems unapproachable by all other systems.

Since 1870, a large suspension bridge was built at Pittsburg, and the construction of the East River Bridge in New York has been continued.

Point Bridge.—The first, constructed in two years (1873-1875) carries a double track and two side-walks. The floor has a width of $10^m.37$ between centers of rails. The length of the principal span (center to center of piers) is, as I have already indicated, 244 meters. The height left under the bridge at low water is $24^m.40$. The suspension chains are not formed of wires as in the Roebling bridges. They are composed of bars

$6^{m}.25$ long side by side, and united at their extremities to the next bars by a pin or hinge of $0^{m}.15$ diameter.

The method employed to give the bridge great stiffness consists mainly in the fact that the chains are connected with the floor by posts not susceptible of flexure, while each half chain acts as the lower chord of an inclined truss, of which the upper chord descends in a straight line from the summit of the tower to the center of the floor. Posts and diagonal bracing all jointed unite the two chords. The upper chord, formed of channel bars and plates, gives a rectangular section of 0^{m} 56 width and 0^{m} 33 height. The distance between the two chords is 6^{m} 71.

The East River Bridge.—The two piers and the abutments of this bridge were completed in 1873, but the continuation of the work has been subjected to long delays on account of financial difficulties. The cities of New York and Brooklyn have now charge of the enterprize instead of the original company.

The superstructure is in course of erection. A section of one of the principal cables can be seen at the Exposition. It is formed of 6000 cast steel wires galvanized. Its diameter is about $0^{m}45$. Its resistance to rupture is estimated at 10 tons.*

II. FOUNDATIONS.

Foundations by compressed air, invented in France in 1841 by M. Triger and applied ten years later to the reconstruction of the Rochester Bridge, were introduced in America in 1855. They have been notably applied there, on a scale unknown in Europe, to the two great bridges at St. Louis and New York.

Among other differences one remarks,

First. The immense area of the caissons (reaching nearly 16 arcs in the New York Bridge).

Secondly. The substitution of massive timber work for iron in the caisson of this same New York bridge, where the thickness of the platform reached 7 meters.

Third. The employment of a machine called a sand pump in the St. Louis bridge.

Fourth. The great working depth ($33^{m}.70$) under water which was reached in this same bridge.

Fifth. Lastly, and above all, the location of the air chambers at the bottoms of the wells.

Attacked on a grand scale from the end of 1869, the foundations of one of the piers of the St. Louis bridge were far advanced when I visited there in September, 1870. I was very much struck with the air locks which, instead of being placed above the level of the water and removed whenever it became necessary to lengthen the wells of access to the working chamber, were established as a fixture in the chamber itself. The descent was thus made in a central well in the ordinary atmosphere by means of a large cage 3 meters in diameter. In the well *a circular staircase was built*, which later gave place to an elevator.

The descent was thus made to a point 6 feet below the ceiling of the caisson, then by a door on a level with the air chambers, which was two meters in diameter. When the equilibrium of pressure was once established the outside door opened itself, and nothing more was to be done but to jump to the earth from a height of about $0^{m}.80$. In air thus strongly compressed it became at least necessary to reduce the time of labor to less than an hour. What an advantage not to be obliged to subtract the time necessary to ascend and descend a height equivalent to 10 stories of a Parisian house. What relief to the laborers, generally overcome with fatigue and reeking with prespiration at the end of their task. What convenience for the transmission of orders, for the introduction of tools, in fact for all kinds of communications. Besides the space necessary for the compressed air was much reduced, and it was no longer necessary to construct the portion of the wall situated above the chamber of heavy plate iron.

It was sufficient that it should be protected from any water which might filter through the masonry by an external skin of iron, or much more economically (as was done by Mr. Eads in 1870) by an interior lining of pine wood staves. The publicity given in 1873 to the description of the St. Louis bridge had the effect to bring forward the fact, that in the construction of the Cologne bridge over the Rhine an air chamber had been placed

* *Note by Translator.* Perhaps 10,000 tons is meant.

permanently in the working chamber as at St. Louis. This idea was elaborated jointly in 1869 by M. Masson, builder, and M. Sadi Carnot, *ingénieur des ponts et chaussees.*

It was carried out in the Spring of 1870 under an effective head of water of about 8 meters. This work is represented to-day with the rest in the French section of the Exposition, in the pavilion of the Minister of Public Works.

The reputation of M. Masson will lose nothing by the simultaneous execution of the Cologne and St. Louis bridges. While M. Sadi Carnot should be doubly happy at the importance which the invention to which he contributed has taken in America; an invention carried out perhaps at the right moment, by an intelligent breeze from the banks of the Rhone to those of the Mississippi.

CONCLUSION.

Of the facts before mentioned, the following concluding summary may be made:

Americans have proved that it is possible to construct readily bridges of 100 meters and more of span. In countries where the need is not felt in a similar degree, one should at least conclude from these examples that it is best to look twice before constructing bridges which may impede navigation and prove obstacles to the flow of freshets.

As regards river bridges we can (at least in the present state of the question) retain riveted lattice bridges and leave to Americans those pin bridges of large lattice which they have, nevertheless, made so useful. But all nations can learn from America useful lessons on the art of compressed air foundations.

In that country where the construction of railways is nearly completed, government and public opinion return, after 40 years of interval, to navigable routes. They are occupied to-day not only in creating great lines where they did not exist before, but also with improving and completing existing lines. When traffic is abundant a good navigable line will transport freight much cheaper than a good railway. They are the natural counterweights to the omnipotence of railways and the most efficacious regulator of bulky and heavy freights. To that end, the Federal Government is studying with much care improvements relating to the system of interior navigation. It is itself executing works whose expense can be estimated with exactness and whose results are certain. But it leaves to local interest and private industry the responsibility of adventures similar to those which are being accomplished at the mouth of the Mississippi.

While the government abstains here, Mr. Eads and his associates are going ahead and each is in his place. We can but approve the reserve of the government, but how can we help admiring Mr. Eads, who is wrestling with the bar of the Mississippi, foot by foot, and giving blows which recall those of the gallant warriors of Charlemagne. Let us hope for him, that the extremities of his jetties will neither be carried off by tempests nor engulfed by moving sand, that his mattresses will be imbedded in mud before being devoured by the *teredo navalis.* Lastly, that the bar—temporarily put to flight, will not reappear before the time appointed by its undaunted enemy. Whatever the definite result of this gigantic hydraulic river experiment may be, it has a right, it seems to me, to the sympathy and attention of the engineers assembled at the Exposition, who have a common interest in that progress of which their art will always be capable.

THE following gentlemen have been appointed as Commissioners for the purpose of inquiring and reporting whether, with respect to the influence of fluctuations of atmospheric pressure upon the issue of fire-damp from coal, to the adoption and efficient application of trustworthy indicators of the presence of fire-damp, and generally to systematic observation of the air in mines, to improved methods of ventilation and illumination, to the employment of explosive agents in the getting of minerals, and to other particulars relating to mines and mining operations, the resources of science furnish any practicable expedients that are not now in use and are calculated to prevent the occurrence of accidents or limit their disastrous consequences:—Mr. Warington W. Smyth, F.R.S., Sir George Elliot, M.P., Mr. F. A. Abel, C.B., Mr. Thomas Burt, M. P., Mr. Robert Bellamy Clifton, F. R. S., Professor Tyndall. F. R. S., Mr. Lindsay Wood, and Mr. William Thomas Lewis.

TESTING IRON BY ELECTRO-MAGNETISM.

By A. HERRING, Mining Engineer.

Written for Van Nostrand's Magazine.

[The theory of this process which is believed to be new is briefly this: When a bar of iron or steel is passed through a helix which is traversed by a current of electricity, the bar becomes a magnet of greater or less intensity depending upon the current, the coil of wire, and the mass of metal. If the metal be homogeneous the polarity of the bar or plate may be inferred from known laws of electro-magnetism. If, however, there be a want of continuity of fibre at some point, new poles are established there which a properly-mounted needle will readily detect. But such a condition of the metal constitutes a *flaw;* hence the usefulness of the method.

The details of the testing process and of the apparatus employed are described by the inventor as follows.—Ed.]

Any inequality in density, the slightest crack, flaw, or any molecular change tending to disturb the homogeneity of the iron, is a false magnetic pole the intensity of which is in direct proportion to the extent of the defect. By my process, I locate and measure the intensity of the false poles of any piece of iron, and estimate the extent of the imperfec tions causing such false poles.

A magnetometer sufficiently sensitive for this purpose is a nicely balanced dipping needle. These needles I have made of different forms and sizes to suit them to the work for which they are intended. In testing iron, it is not necessary that it should be magnetized to saturation, but the inducing current should be constant and of uniform intensity. To magnetize the iron to be tested, a coil or helix is used constructed of insulated copper wire, of such size and length as will magnetize it with the least amount of battery power, and the helix of such dimensions as to admit of the iron being passed through it.

To test steel wire 0.20 of an inch in diameter the helix should be composed of about six hundred feet of No. 20 copper wire, insulated and wound in superposed layers upon a spool of rubber or other non-conductor, which should be about two and a half inches in length, with an aperture, one-fourth of an inch in diameter, through it. The helix should be made stationary upon a frame or table. A wooden block about 3 inches in width, with a hole one-fourth of an inch in diameter through it, is then placed in front of the helix in such a position as to allow the wire to pass through both helix and block, and upon this block the dipping needle is to rest, about one-eighth of an inch above the aperture through which the wire is to be passed. The needle to be used is one inch in length, with its axis in the center, and to the negative pole is attached an aluminum pointer, to move over the arc of a circle 4 inches in diameter—the arc divided into degrees—the needle so mounted that when the positive pole is down the pointer shall be at zero on the arc, which is numbered right and left from zero to 90 degrees. This, with one cell of a Watson's battery, and two steel test wires, which I will describe below, completes the apparatus necessary for testing the wire. These test wires should be of the best homogenous steel, about two feet long, of the same size as the wire to be tested, the breaking strain and density of each, known from actual test. No. 1 test wire should be thoroughly annealed, and No. 2 evenly tempered to that degree of hardness at which it will bear the greatest strain.

With a coil of wire to be tested wound on a reel or drum in such a position as to be passed through the helix and block and wound on a second drum provided for the purpose, the battery is charged and carefully tested by means of a galvanometer, and its strength noted, we are ready to begin the test. The battery is connected with the helix so as to bring the positive pole of the helix next to the block on which the dipping needle (with its pointer and arc) is placed. No. 1 test wire is then inserted in the helix and block and drawn through until its center is within the helix. The magnetometer or dipping needle is then placed on the

block immediately over the wire, the block is then moved to and fro along the wire until the pointer rests at zero on the arc, the block and magnetometer are then so fastened that the distance between them and the helix shall remain the same throughout the operation of testing. Test wire No. 1 is now removed and No. 2 put into the helix and block. This will cause the positive pole of the needle to rise in the direction of the helix. When it has ceased to vibrate note the position of the pointer on the arc. This indicates the maximum strength and density, and zero the minimum of the wire. Remove the test wire, and pass the end of the wire to be tested through the helix and block, fasten it to the reel or drum on which it is to be wound, and while an assistant winds it from one drum to the other, note the changes of the pointer, marking the defective places—the defect being in proportion to the variation of the pointer between the maximum and minimum as found by the test wires. Should the pointer pass zero, it indicates a flaw in the wire at the point then passing under the needle. This action of the needle indicates an actual break in the fibers of the metal and is to be estimated as in proportion to the deflection of the needle. Practice soon teaches the proper estimate to place on such defects. By this process we estimate the comparative magnetic intensity of the wire and the tensile strength of every foot of it, without impairing it in the least.

The testing of boiler plates is done in a similar manner, or upon the same principle. The plate is placed in a flat helix, connected with sufficient battery power to magnetize it, and passed under a number of magnetometers or needles arranged with their axis all in a line at the proper distance in front of the helix. These needles must be all of the same size, shape, and weight, and must be magnetized to the same intensity, and, for convenience in operating, may be mounted on a strip of wood or brass of sufficient length to extend across the plate to be tested. They should be placed about five inches apart, and free to rise and fall in the direction of the helix, and each supplied with a graduated arc by which to note the variations of the needles.

So, in testing iron colums, large shafting, heavy machinery and ordnance, as well as small articles such as pieces of fire-arms, sabres, etc., the apparatus is only modified in shape and intensity to adapt it to the size, weight, and shape of the piece to be tested, embodying the same principles and same laws in all tests. Though the form, size, and magnetic intensity may be changed according to the particular work to be done, we accomplish the same result in all, namely, to point out with certainty the exact point or spot where any flaw or imperfection exists, and also indicate the amount or extent of such defect.

In Figs. 1, 2, and 3 is represented an apparatus adapted for testing the density and tensile strength of steel wire, in which a foundation, as at Z, constitutes a support for the main portion of the apparatus, which is represented in connection with a cell, of what is known as "Watson's battery."

The essential parts of the apparatus consists of drums or reels A and B, a helix, C, a magnetometer, D, mounted and movable upon a scale, E, a frame, F, and a battery.

The helix C rests upon a projecting portion, *z*, of the foundation Z, and is in the form of a spool, the body portion *c* of which is made of hard rubber with an aperture through it from end to end. Upon this spool insulated copper wire *c'* is wound in layers, the ends of which wire are connected to the binding-screws *n* and *p*.

The magnetometer D consists of a magnetized needle or bar of steel, *i*, the negative end of which has been lengthened by the addition of a piece of aluminum, *i'*, in order that the operator during the act of "testing" may read on an enlarged semicircle, *m*, as shown clearly in Fig. 3. The magnetometer D is seated upon a thin metal plate, *d*. This plate has its rear portion or edge bent over and under the rear edge of the scale E, while front portions, as at *d'*, are in like manner fitted to or clasped upon the front edge of said scale, thus retaining the magnetometer in position upon the scale, and also allowing it to be moved longitudinally thereon. A portion of the plate *d* is cut away between the lapped parts *d'*, so that the scale E is exposed to view; and as the magnetometer is moved upon the scale

Fig. 1.

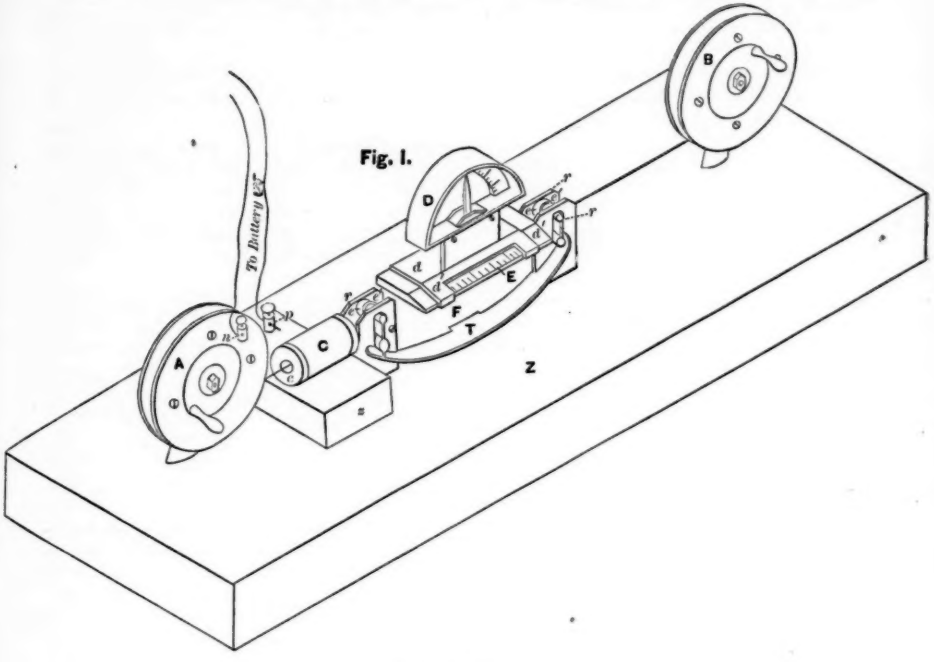


Fig. 2.

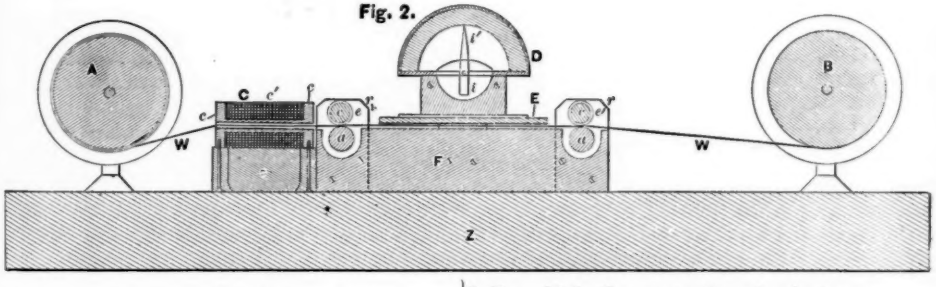
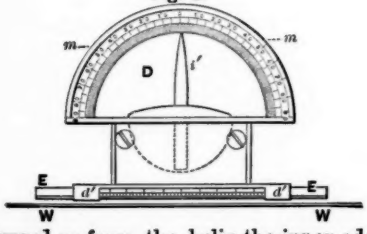


Fig. 3.



toward or from the helix the inner edge of the parts *d'* serve to register with the divisions marked off upon the scale, and thus indicate the exact distance from the helix which the magnetometer is to occupy at the commencement of a "test," and which position throughout the test it must maintain. In other words, the magnetometer is so mounted upon the scale E as to allow it to be moved toward or from the helix C, and admit of its being fixed at any desired distance from the helix to suit the different sizes of wire to be tested.

The frame F is constructed at each end with a set of slotted posts, *r r*, and between these posts are two upper rollers, *e e*, having a groove, *e'*, central of their length to receive and guide the

wire *w* while being tested, and also to fellow ungrooved lower rollers, *aa*. The axles of each of these rollers play loosely (up and down) in the vertical slot of the posts *r*. The axles of the lower rollers extend some distance outside of said posts, so as to engage, as indicated in Fig. I, with a spring, T, attached to the frame F, as shown, so that the force exerted upward by the spring will press the wire *w* firmly against the rollers *e e* in their grooves *e'*, thus keeping the wire at a fixed distance from, but near to, the needle *i* while being wound from the reel A to the reel B.

Operation: To practically test steel wire which is from four to five millimeters in thickness, the apparatus should have the following proportions; Drum A should be of such dimensions as to receive the coils as they come from the manufacturers. Drum B should be about two meters in diameter. The helix C should be composed of about two hectometers of No. 14 insulated copper wire, wound in superposed layers upon a spool, the center of which should be made of a non-conducting substance (such as rubber, paper, or hard wood,) with a hole six millimeters in diameter through the length of it. The scale E should be two decimeters in length, and divided into millimeters. The guide-rollers *e e* and *a a* in frame F should be about three decimeters apart. The spring T should be sufficiently stiff to keep the wire *w* firmly to its place in the grooves *e'* of the rollers *e*. Any form of galvanic battery may be used; but the Watson battery, owing to its constancy, seems best adapted to the purpose. Three cells of this battery should be coupled together and connected to the helix C by the binding-screws *p n*. The arc of the magnetometer should be half of a ten-centimeter circle. The arc should be divided, as indicated in Fig. 3, into degrees, or one hundred and eighty equal divisions, with zero at the top, and numbering right and left to ninety. The needle of the magnetometer should be formed of two pieces: first, a steel bar or needle, four centimeters in length, with a pivot in the center; second, an aluminum point, riveted to the negative end of the steel needle, making the length of the needle seven centimeters. In this manner we bring the axis of the needle near the iron to be tested, which insures the greatest deflection from any change in the magnetic intensity of the iron, and gives an extended arc, *m*, on which we are the better able to read the changes of the needle.

To adjust the apparatus for use, the battery C being in operation, a piece of soft-iron wire, perfectly made, five millimeters in thickness and one meter in length, and of known tensile strength, is placed in the helix C and drawn through until the center of the length of the wire rests within the helix. The magnetometer D is then moved along the scale E until the negative end *i'* of the needle points to zero. This will take place when the magnetometer has been moved toward or from the helix, as the case may be, until the positive end of the needle is directly over the point of greatest magnetic intensity developed in the wire. The soft-iron wire is then removed, and the magnetometer remains throughout the test in its position on the scale E which it now occupies.

In place of the soft-iron wire a piece of steel wire of like dimensions and known strength, and perfectly made, is now in like manner placed in the helix. If the steel is of greater tensile strength than the iron, the pointer *i'* of the magnetometer will fall to the right of zero. Take a note of the point on the scale of the arc *m* at which the pointer comes to rest; remove the steel wire; fix the coil of wire to be tested on the reel A; pass one end of the wire through the helix C and between the rollers *a a* and *e e* in the groove *e'* of the rollers *e*, and then attach the end of the wire to the reel B. Now, turn the reel B, and while the wire is wound from the reel A to the reel B note the position of the pointer *i'* upon the scale of the magnetometer, and calculate the strength of the wire from the ratio found between the two test-pieces. For instance, if the breaking-strain of the soft-iron wire which fixed the needle at zero is two thousand pounds, and the steel wire which fixed the needle, say at forty-five degrees, breaks at four thousand eight hundred pounds, the increase in pounds per degree is sixty-two pounds, and the ratio found, which is thirty, will serve to calculate the strength of any portion of the wire being tested, unless the pointer *i'* passes zero to the left,

which indicates an actual flaw in the wire, and the greater the deflection of the pointer to the left of zero the greater the flaw.

In testing large shafting, bars of metal, heavy ordnance, pieces of machinery, and small articles, such as pieces of fire-arms, sabers, bayonets, &c., the apparatus will of course be adapted in intensity to the size, weight, and shape of the piece under examination.

Fig. 4.

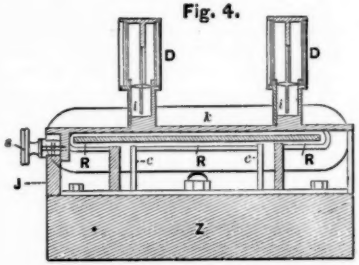


Fig. 4 represents the helix as constructed and arranged for testing plates of iron and steel. By this construction and arrangement we are able to test rapidly and give the density and tensile strength of every square inch of the largest-sized boiler or armor plates. The helix C is flattened or elongated, so as to admit the metal plate R to be passed through it. This helix is composed of superposed coils of insulated copper wire the ends of which connect with binding-screws *n* and *p*. The magnetometers D D are mounted on a movable bar, *k*, that they may be the better moved to and fro in front of the helix, and fixed at any desired distance from it by means of the set-screw *s* to the scale J. In the figure, R indicates a plate of iron or steel position in the act of being tested, the plate resting upon trunnions *c c*, over which it is drawn by a drum.

To test a plate the helix and battery should be suited in size and power to the thickness of the iron or steel to be tested. The movable bar *k* should reach from side to side of the plate, and have fixed to it a number of magnetometers about two decimeters apart. These magnetometers should be of the same construction as that described for testing wire. Connect the poles of the battery with the helix C by means of the binding-screws *n* and *p*, so as to magnetize the plate with the negative pole to the right of the helix; pass the end of the plate through the helix and fasten with clamp, and while an assistant draws it steadily through the helix note the changes of the needle *i* in the several magnetometers.

Having previously ascertained the magnetic intensity of a cubic centimeter of iron and steel of known density and tensile strength, we are enabled to calculate the strength of any part of the plate by a comparison of its magnetic intensity.

Mr. Herring's process is protected by letters patent under date March 11th, 1879.

At a recent meeting of the French Academy of Sciences M. de Lesseps gave an account of the Suez Canal currents furnished by M. Lemasson, the company's engineer, the result of observations taken since 1871 at Port Said, Suez and other stations. Notwithstanding Lake Timsah and the Bitter Lakes, which form two great regulators, the north and south branches of the canal are not unaffected by each other. From May to October the winds cause a rise of level at Port Said and a fall at Suez. This difference of level, which reaches about 15.5 inches, leads to a current from the Mediterranean to the Red Sea—a current which, though interrupted by the tides, drives a considerable volume of water from north to south. In winter, on the contrary, the high winds give the Red Sea a level higher by about 12 inches than the Mediterranean, causing a current from south to north. About 14,126 million cubic feet or about 400,000,000 tons of water yearly pass from one sea to the other. This, coupled with the tides, tends to neutralize the effects of evaporation from the surface of the lakes and to dissolve the basis of salt in the Bitter Lakes. That basis was 32.8 feet in thickness, but is gradually dissolving, especially in the course of vessels. The velocity of these local currents varies between Port Said and Lake Timsah from 0.5 feet and 1.3 feet per second; while in the broader part, between Suez and the Bitter Lakes, it is from 2 feet to 3.6 feet per second.

THE PRISMOIDAL FORMULA.

By J. WOODBRIDGE DAVIS, C. E.

Written for VAN NOSTRAND'S MAGAZINE.

PREFACE.

It would be difficult to estimate the value of the prismoidal formula, or to enumerate the branches of business and professional work in which it is constantly employed. Its advantage over most other rules for mensuration, lies in the fact that its extreme simplicity is coupled with a wonderful generality, even as an exact rule, while it approximates very closely the contents of a host of shapes. It is, therefore, a rule which can be understood and applied by every intelligent person who is not acquainted with the more general, but less simple, rules of higher mathematics, at the same time that it is a rule applicable to a great many of the most common shapes.

But, apart from its immense importance as a practical rule, the prismoidal formula has always been of great interest to mathematicians, and, it may be said, a mystery, on account of its wide range of application. Perhaps everyone, after studying its derivation from the rectangular prismoid, has been startled when first he saw, or heard it announced, that it applies to the sphere and the ellipsoids. Every author of a treatise on mensuration takes delight in demonstrating its applicability to the three round bodies, and in deriving from it numerous special rules. The limits of its applicability have been gradually widened by successive discoveries—often exciting great surprise—of its applicability to shapes before mensurated by more difficult rules, or merely approximated for want of convenient formulæ. There has also been much speculation as to the number and variety of prismoidal shapes, and as to the nature of their connection.

The object of this article is to establish a single formula that shall indicate exactly the limits of the field of application of the prismoidal rule, and shall express the relation which exists between all prismoidal spaces; also, to show the value of this formula as a criterion in determining whether or not the rule apply to given spaces; and, finally, to convey an idea of the vast extent governed by this rule, and to describe, in illustration thereof, several extraordinary shapes more curious than available.

It must be confessed that the writer was almost discouraged from publishing this matter by reading, after his own work had been accomplished, two excellent articles on the same subject, of which mention is made in proper place. There are, however, so many points of distinction as to have induced the writer to persevere. Moreover, the articles referred to were published in mathematical journals, one extinct, and the other, presumably, not widely circulating among engineers.

This paper is condensed from a thesis, bearing name of *Prismoidal Formula*, and deposited in the Engineering Department of School of Mines, Columbia College. In accordance with present design, several of the conclusions, which are merely interesting, perhaps astonishing, are simply stated. For demonstration, if any require it, the reader is referred to that work. Also, three rather extensive notes have been entirely omitted. These are: *Broken Lines represented by Equations of Infinite Degree; Indeterminate Coefficients*, in which the ordinary proof contained in the algebras is criticised; and *Limits to the Number of Roots of Transcendental Equations*. Two chapters, containing discussions on *A New Center of Gravity Formula* and *A New Moment of Inertia Formula*, have likewise been excluded; but, as these latter are the most practical portions of that work, the writer hopes to be able soon to present them in acceptable shape.

PRISMOIDAL FORMULA.

Space, whether illimitable or contained by a certain boundary, may be generated by a right plane, always remaining parallel to two non-parallel straight lines and moving in a direction perpendicular to them, the motion being constantly progressive, and the boundary of the plane at every position being its intersection at that position with the boundary of the space. Thus, the generatrix

passes once through every point of the space, and is, accordingly, said to describe it.

The boundary of the space may be however intricate, and possess any number of convolutions or entirely separate parts.

Letting y represent the area of the generatrix at any position, [when the space is of two dimensions, y is the length of generating line], and letting x denote the distance of generatrix from first limit, we have, when y everywhere between those limits varies according to the same function of x,

$$y=f(x), \tag{1}$$

and for the amount of space, or volume,

$$v=\int_0^l f(x)dx. \tag{2}$$

(1) may be considered the equation of the space, since it indicates the magnitude of the generatrix at every position. It is independent of the shape of boundary. Therefore, the space, whose equation is (1), may be contained by any of an infinite variety of boundaries; and its volume for each is the same.

[In the single case where the space is a plane, and one longitudinal boundary a straight line, (1) is the equation of the other longitudinal boundary as well as of the space].

If, however, between the limits y vary as $f_1(x)$ to a distance m_1, then, abruptly, begin to vary as $f_2(x)$ to the distance m_2 from origin, and so on change the mode of its variation till the second limit be reached, then the equation of the space becomes

$$y=f_1(x)_0^{m_1}, f_2(x)_{m_1}^{m_2}, \ldots f_n(x)_{m_{n-1}}^{l}; \tag{3}$$

and

$$v=\int_0^{m_1} f_1(x)dx+\int_{m_1}^{m_2} f_2(x)dx\ldots. +\int_{m_{n-1}}^{l} f_n(x)dx. \tag{4}$$

The manner of variation of y may be such that it can be represented between given limits by a succession of known functions, only when their number is infinite.

But, however a generatrix may vary in a given space between limits, it must, at a distance x_1, have some magnitude, which let y_1 express.

At other distances, x_2, x_3, $\ldots. x_{n+1}$, let the magnitudes of y be represented by $y_2, y_3, \ldots. y_{n+1}$. In the equation of the nth degree,

$$y=A+Bx+Cx^2\ldots\ldots+Kx^n, \tag{5}$$

the $n+1$ coefficients may have such values as to satisfy $n+1$ conditions concerning x and y. Let these be the conditions that when $x=x_1$ y shall be y_1 etc. Then (5) is the equation of a space, which coincides in magnitude with the given space at the distances x_1, x_2, etc. from the origin. If every position of the generatrix be considered, (5) is an equation of infinite degree; and it is the equation of space which coincides in magnitude with the given space at every distance from the origin. Therefore, the generatrix of every space, between any limits, varies according to a single function of its path; and that function is represented by an algebraic expression of infinite degree, in terms of the path, with integral, positive exponents and constant coefficients. If we denote this function by F (x), the equation of every space is

$$y=F(x).* \tag{6}$$

The volume of a space, to which the prismoidal formula applies, is

$$v=\frac{l}{6}\left\{A+4M+B\right\}, \tag{7}$$

where l is the distance between limits,

* In note 1 of the thesis mentioned in preface are given examples of broken lines represented by infinite equations. Among them are the perimeter of square, which, as a continuous line, referred to axes parallel to its sides and intersecting at its center, is represented by

$$x^\infty+y^\infty=r^\infty,$$

when ∞ is even; and the axes themselves, within square, as one continuous line, represented by the equation

$$x^{-\infty}+y^{-\infty}=r^{-\infty},$$

when the degree is even.

$$x^\infty+y^\infty+z^\infty=r^\infty$$

is the equation of the surface of a cube, and

$$x^{-\infty}+y^{-\infty}+z^{-\infty}=r^{-\infty}$$

is the equation of co-ordinate planes within cube, when ∞ is even.

The following also is quoted from this note.

The fact that any irregular shape can be represented by an equation, allows us to use, in treatises on mechanics, the expression dv to denote the content of an infinitesimal section of that shape; also, to use the other differential and integral symbols pertaining to discussions of center of gravity, moment of inertia, etc. Demonstrations, made with aid of these symbols, and the processes they represent, are generally more concise than those wherein the expressions of mere summation of independent particles are employed. When the general principles are applied to known forms, the symbols of the calculus must be used, because they indicate how the summation can be effected. Hence, for uniformity also, it is advantageous to use latter symbols even in general cases.

and A, B, M, are the magnitudes of the generatrix at first and last limits and at the position midway between.

The extent of application of the prismoidal formula may be determined by considering it in connection with equation (6), since this represents all spaces. The F (x) is the sum of any number of algebraic expressions, each of the form Kx^n, n being positive and integral. The prismoidal formula applies to the space represented by one of these, when

$$\frac{l}{6}[K(0)^n+4K(\tfrac{1}{2}l)^n+Kl^n]-\int_o^l Kx^n dx=o$$

or, $$\frac{Kl}{6}\left\{o^n+\frac{4l^n}{2^n}+l^n\right\}-\frac{Kl^{n+1}}{n+1}=0. \quad (8)$$

This equation is true when the coefficient K is zero, or when l is zero, in either case when the volume is zero. The former is the case of a line or number of lines, of any complexity between any limits. The latter is the case of a generatrix of any magnitude maintaining a single position. The equation is also true when n has any of the values 0, 1, 2, 3.

For all values of n, except zero, the first term in the brackets is zero, and the equation may have the more convenient form

$$Kl^{n+1}\left\{\frac{2^{1-n}}{3}+\tfrac{1}{6}-\frac{1}{n+1}\right\}=0. \quad (9)$$

When n exceeds 5, the first member of this equation is always numerically greater than zero, and, consequently, the equation is false. Also, when n is either 4 or 5, the equation is false. Hence, 0, 1, 2, 3, are the only positive integral values of n, such that the prismoidal formula applies exactly to the space represented by the equation $y=Kx^n$.

Applying now the prismoidal formula to the space $y=F(x)$, and subtracting from the volume thus found, the true volume found by integrating each term, the difference, equated with zero, we find to be

$$\left.\begin{aligned} &K_1 l^{m+1}\left\{\frac{2^{1-m}}{3}+\tfrac{1}{6}-\frac{1}{m+1}\right\}\\ &K_2 l^{n+1}\left\{\frac{2^{1-n}}{3}+\tfrac{1}{6}-\frac{1}{n+1}\right\}\\ &K_3 l^{p+1}\left\{\frac{2^{1-p}}{3}+\tfrac{1}{6}-\frac{1}{p+1}\right\}\\ &\qquad +\text{etc.}\end{aligned}\right\}=0. \quad (10)$$

If the F (x) consist of terms affected by no other exponents than 0, 1, 2, 3, [when an exponent is zero the wanting term in enclosed factor must be supplied] all the enclosed factors are zero. Hence, the prismoidal formula applies between any limits to the space whose equation is

$$y=a+bx+cx^2+dx^3. \quad (11)$$

If the F (x) include terms having greater exponents than three, then, in eq. (10), the terms corresponding to the exponents 0, 1, 2, 3, are zero; and, if q denote the number of terms remaining, the truth of equation (10) depends upon the values of $2q+1$ independent quantities, viz: the coefficients and exponents of the terms and the distance between limits. Each, consequently, must be a certain function of all the rest, or one of a definite number of particular functions, in order that this equation may be true.

Thus, from equation (10) we see that for given values of the coefficients and exponents, that is, for a given space, there are as many values of l such that the equation shall be true, as the number indicating the degree of F (x) and one more. As many of these are zero as the number indicating the degree of the lowest term above the fourth term in the F(x), and one more. The other values may be positive, negative and imaginary. For instance, the prismoidal formula applies to the space

$$y=a+bx+cx^2+dx^3+ex^4+fx^5+gx^6 \quad (12)$$

when $$\tfrac{1}{12}el^5+\tfrac{1}{48}fl^6+\tfrac{23}{672}gl^7=0. \quad (13)$$

Solving this equation in favor of l, we find that besides five zero values

$$l=-\frac{7f}{23g}\pm\frac{21}{46g}\sqrt{\frac{35f^2-92eg}{315}}.$$

If the equation of the space be of infinite degree, then eq. (10) is infinite, and there are an infinite number of lengths of the space subject to the prismoidal formula. If these lengths be none of them negative or imaginary, and if they all be consecutive values of l, then this equation of infinite degree represents a space subject to the p. f. [prismoidal formula] between any limits. This is the only possible case of a prismoidal space of higher degree than the third. But, since for every value of l the value of the functions of l expressed in first member of eq. (10), is zero, this is

the equation of a line coincident with the axis of X. Whatever be the coefficients of this equation, they are equivalent to those of the following equation:

$$y=(y)+\left(\frac{dy}{dx}\right)x+\left(\frac{d^2y}{dx^2}\right)\frac{x^2}{2}+\left(\frac{d^3y}{dx^3}\right)\frac{x^3}{6}+\text{etc.}, \quad (14)$$

wherein y represents the first member of eq. (10), and (y), $\left(\frac{dy}{dx}\right)$, etc., represent the values of y, $\frac{dy}{dx}$, etc., when $x=o$. The last member of eq. (14) is identical with the first of eq. (10), the coefficients having merely a different form. Since the line represented by either equation is coincident with the axis of X, it is tangent thereto at the origin. Consequently, as many successive coefficients must be zero as the number of succeeding points at which the line still coincides with the axis of X. Because every succeeding point is common to both lines, every coefficient of eqs. (14) and (10) is zero. Now, the factor in brackets, of every coefficient of eq. (10), corresponding to a term of fourth or higher degree in F (x), is not zero. Therefore the K of this term must be zero. On the contrary, the factor in brackets, of each coefficient of a term in eq. (10), corresponding to a term in F(x) of third or less degree, is zero. Therefore, the K of any of these terms may have any value. This reduces the equation of every space, $y=\mathrm{F}(x)$, to the same case expressed by eq. (11).*

Between given limits there may be an infinite number of spaces, whose generatrices vary according to algebraic functions of any degree except the fourth, yet such that the p. f. shall apply to each space between those limits. For a given length, l, measured from origin, the p. f. applies to a space, whose generatrix varies as an algebraic function of sixth degree, eq. (12), when, as derived from eq. (13),

$$e=-120\left(\frac{fl}{48}+\frac{23gl^2}{672}\right)=-\tfrac{5}{2}fl-\frac{115}{28}gl^2.$$

The conditions, $l=o$, $k=o$, since they are practically useless, being now neglected, and, also, all those spaces to which the p. f. does not apply between every pair of limits, there remain, as spaces to which the p. f. universally applies, of all spaces, those only which are represented by the equation

$$y=a+bx+cx^2+dx^3. \quad (11)$$

Eq. (11) is independent of shape of boundary. It may be used, as a criterion, to ascertain whether the p. f. apply or not to spaces of given boundary. It only needs that the general expression for magnitude of generatrix in terms of path be determined. If this be an algebraic expression of degree not higher than third, the formula applies. Otherwise, the formula does not apply. This may also be used as a method of demonstrating that the p. f. does apply. Its application to either case saves us the labor of actually applying the p. f., and, then, some other rule known to be correct, and, finally, comparing results. Thus, eq. (11) serves to generalize and abbreviate greatly the demonstrations of rules for mensurating common solids. To the practical calculator it is both a guide and a guard.

Another use, perhaps not very practical, that can be made of eq. (11), is described as follows. By means of eq. (11) may be obtained expressions for boundaries, which shall satisfy any number of given, non-conflicting conditions, yet shall contain spaces, all subject to the prismoidal rule. In this manner, by imposing unusual conditions, prismoidal spaces of most extraordinary shapes, may be discovered.

Portions of prismoidal space may be negative. Such portions are generated by negative generatrices. For instance, the quadrilateral, represented in Fig. 1,

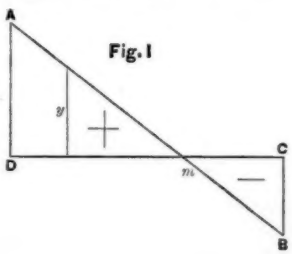


Fig. 1

* It will be noticed that the result arrived at in this paragraph might have been directly quoted from the chapters on *Indeterminate Coefficients* in treatises on algebra. But, as the demonstrations in these seemed to the writer imperfect, (for reasons discussed in note 2 of thesis mentioned in preface) he has preferred to use the proof shown in text.

is generated by the moving line, y. This becomes smaller, and, consequently, the infinitesimal differentials of the area become smaller, as it moves to the right. At the point m it is zero; and here its motion adds nothing to the space. Proceeding, it diminishes at the same rate, becoming, therefore, negative, and producing negative space. The p. f. applies between any limits, only when the space BCm is negative. When this is positive, the shape is, in fact, not a quadrilateral, but a hexagon, whose sides are Am, mC, CB, Bm, mD, DA.

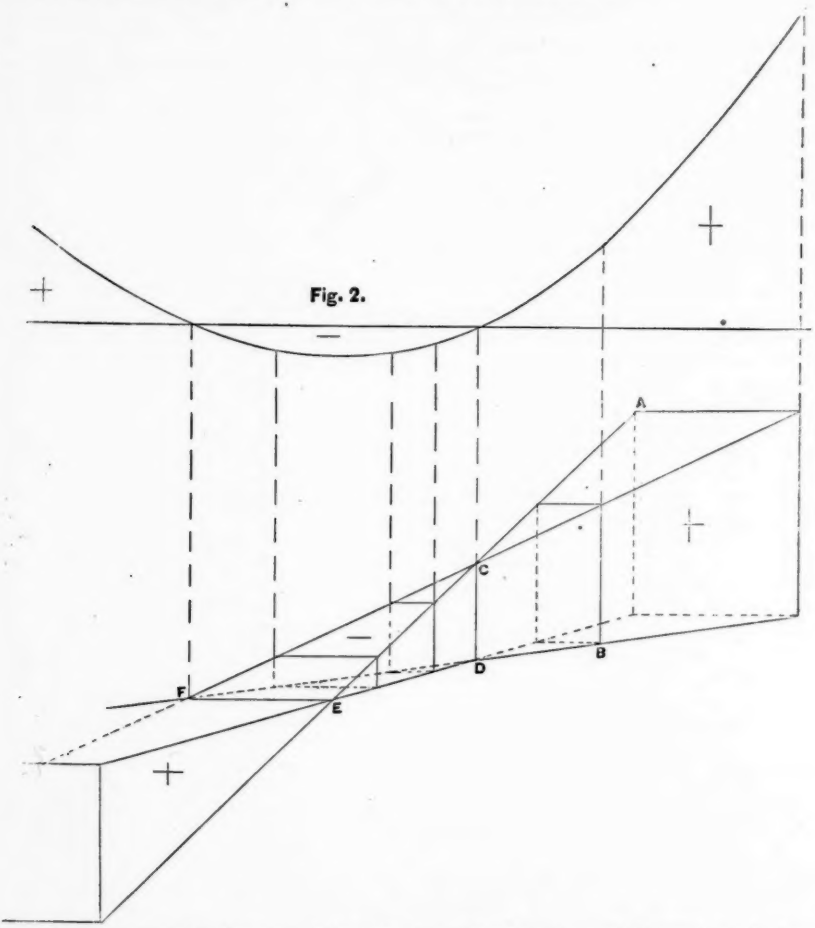


Fig. 2 represents the complete prismoid. This may be defined as *The space generated, between any limits, by the motion of a varying rectangle, whose sides maintain a constant direction, while one vertex proceeds along a straight line.*

Both the product, and the quotient resulting from the division of one by the other, of the rectangle's two dimensions, must be functions of the path, and, therefore, variable.

The rectangular pyramid can be defined in same words, except that *the product must be variable and the quotient constant.*

In the prism *both are constant.*

To complete the four cases, we must consider the space generated by a rec-

tangle whose dimensions vary as $\frac{a}{x^n+b}$, x^n+b. In this shape *the product is constant and the quotient variable.*

It follows, from this definition of the prismoid, that one pair of faces must intersect in advance of the remaining pair. At this position the generatrix is zero. The immediate space following is negative, as the product of a positive by a negative dimension. The generatrix becomes zero again when the other faces cross, and, proceeding, becomes permanently positive, as the product of two negative dimensions. The generatrix passes through the same series of changes through which the ordinate of the parabola sketched above passes. The generatrix of the lateral surface passes through the same changes as does the ordinate of Fig. 1.

AB is the conventional prismoid. Both varieties of wedges are shown, as faces cross at the same section. Here the area of generatrix is zero; but immediately on each side it is positive, as the product of two positive or of two negative dimensions. The generatrix of lateral surface passes through same changes as does the ordinate of Fig. 1.

Consider now the triangular warped-faced shape, Fig. 4. It is generated by the motion of a varying triangle, which remains parallel to one plane, while its vertices are constrained to follow any three straight lines. The three faces are hyperbolic-paraboloids.

Referred to the plane through one directrix, as CC′, and the plane of generatrix at any position, as III, the area of generatrix, at a distance x from III, as at also the pyramid CDEF, which, as a prismoid with no end areas, has excited much attention.

The peculiarity of the middle branch of the prismoid is, that the amount of space between any two of its cross sections, as calculated by the trapezoidal rule, is less than the amount by the prismoidal rule, if the space be considered positive. The important fact that there are such sections of a prismoid seems to be unknown to some writers on the applications of the p. f., while others appear to regard it as a singular exception. A conjugate peculiarity of the middle branch is that between any two cross sections thereof, one dimension increases while the other decreases. Considered analytically, *not practically*, it is an universal rule that *between any limits, the amount of space in a prismoid, as calculated by the trapezoidal rule is excessive.*

Fig. 3 is a pyramid. Both pairs of

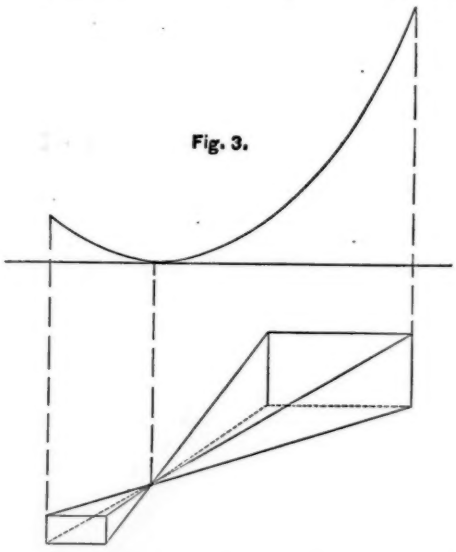


Fig. 3.

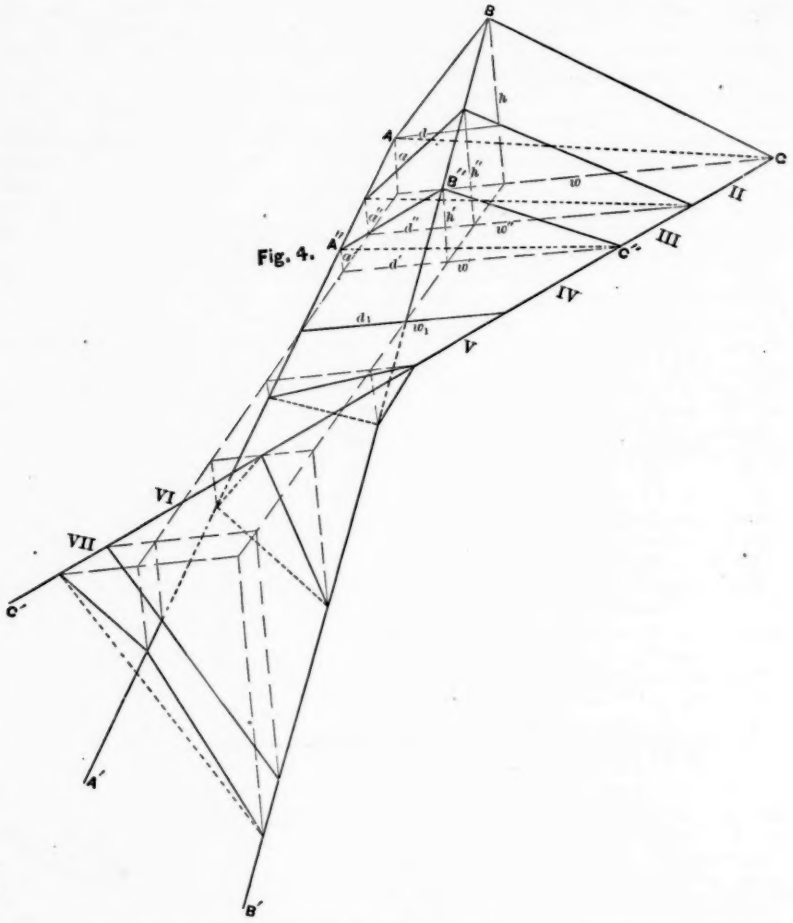


Fig. 4.

II, the measurements of diagram used, is, according to rule B, page 293, of April No. of this Magazine,

$$\tfrac{1}{2}[h''w'' + a''(d'' - w'')],$$

where, in terms of measurements of two given positions of generatrix,

$$h'' = h' + (h - h')\frac{x}{l},\quad a'' = a' + (a - a')\frac{x}{l},$$

$$w'' = w' + (w - w')\frac{x}{l},\quad d'' = d' + (d - d')\frac{x}{l}.$$

Without actually making the substitutions, it is seen that, were they made, the general expression for area of generatrix would appear as a function of x to the zero, first and second powers only. Hence, the p. f. applies between any limits.

The form of the equation of this space,

$$y = a + bx + cx^2,$$

shows that it may contain a negative branch; but, if so, that two consecutive infinite branches are positive. When the generatrix is about to change its sign, it must become zero, or must become a line. [If it become a point, the figure is

a pyramid.] Hence, all the directrices intersect their opposite surfaces, respectively, in one element common to the three surfaces. The latter immediately enclose negative space. [*i.e.*, space on the opposite side of surface. See Fig. 4, page 293, April No. of this Magazine.] But the directrices must again cross their opposite surfaces in one element, which divides the negative from second positive branch.

This leads to the following theorem, which it would perhaps be difficult to prove by other methods.

If three hyperbolic-paraboloids have a common plane director, and a common element at each of three points of which two of the surfaces are tangent to each other; then it is certain that the surfaces have one other element in common, at which they are in like manner tangent to each other; and it is also certain that no two of them can have a third common element, nor any other point of tangency.

If one directrix be tangent to opposite surface, the surfaces are tangent to each other throughout the element, and, consequently, each other directrix is tangent to its opposite face. Hence, the surfaces do not intersect, there is no negative space, and the two elements mentioned in the theorem are consecutive.

Similar theorems, when the directrices are more than three in number, and when they are curves, can be constructed.

This shape is useful as well as interesting. If CC′ be the center stakes of consecutive cross sections of a cutting, AA′ the slope stakes on one side, then AC, A′C′, represent top lines of regular cross sections, and B,B′ are the "breaks," which render the solid irregular. Every irregular solid is compounded of a regular solid and a number of these triangular shapes, positive and negative, which form the ridges and hollows, as seen along BB′. A study of this shape reveals the fact that the error of applying the trapezoidal rule to each of these warped-faced portions may be excessive when compared with that of analogous plane figures; and that it is equal in importance to that produced on the whole regular solid by use of the same rule. Some writers have thoughtlessly neglected these errors, concluding without investigation that they must be trifling, and that the only error worth correcting is that of the regular part of solid. Others have made the errors of irregular part arbitrary functions of the error of regular part, instead of independent functions of their own measurements.

Another peculiarity of this peculiar shape, though important, is not well known. The portions of Fig. 4, limited by the planes I, IV, and IV, V represented the case of a ridge or hollow which fades out at one section. The volume between IV and I is

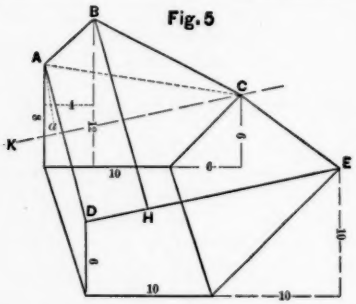
$$[a(2d+d_1)+(h-a)(2w+w_1)]\,\frac{D}{12}.$$

Here it is seen that to give d_1 any increment, m, increases the volume by $\frac{1}{12}\,am$D; while to give d the same increment, increases the volume by $\frac{2}{12}\,am$D. Therefore *the effect of shifting the fading end is half as great as the effect of shifting the other end*, B, *as far in same direction.* Earthwork measurers are very careful to fix the extremity B, but they seldom, if ever, notice the other end. The effect of d_1 on the volume is greater as the opposite face ACC′A′, is more warped. If this be plane, a is zero, and d_1 has no effect.

To illustrate the serious effect the neglect of d_1 has in practice, consider this example of the right side of a railroad solid, which was given by Prof. Gillespie to illustrate his plan of fixing the fading ends. He proposed to establish each of them at a point whose distance from center line should bear the same ratio to the measured distance d_1 of the other end, as the side width w_1 bears to w. The measurements are shown in diagram, Fig. 5. The value of a in this example is found to be 5.1 feet. Substituting this in the formula $\frac{1}{12}\,am$D, and making m 1 foot, D being 100 feet, we find, for the increment to the volume, 42.5 cubic feet, corresponding to an increment of 1 foot to d_1, or 43.4 cubic feet corresponding to 1 foot measured horizontally. The total effect occasioned by moving the point H from the end D of the line DE to the other end E, is, in cubic yards, $43.4\times20\div27=32.1$. This is equal to half the content of the whole solid ABCDEH, when at its least value.

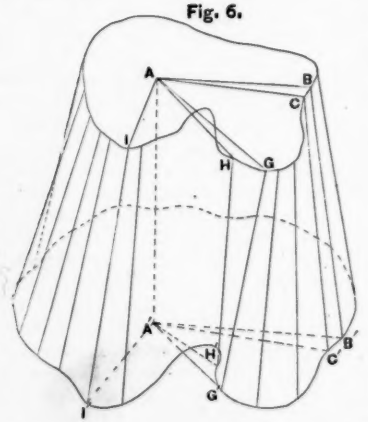
Another use that can be made of this shape [Fig. 4] is to aid in the following demonstration. Consider the space

Fig. 5



shown in Fig. 6, included, terminally, by two parallel planes, and, laterally, by a surface generated by the motion of a right line, always intersecting the planes, and returning finally to its initial position. Assume an axis anywhere, within or without the space, and connect its extremities A,A′, the points of its intersections with the planes, by straight lines with the extremities B,B′,C,C′, of two consecutive elements of lateral surface, as shown in diagram. Denote the area of upper base by E, of lower base by E′, of mid-section by M, and by V the

Fig. 6.



volume of the entire space. Designate the corresponding areas and volume of the space ABCA′B′C′ by dE, dE', dM, dV, and by h, the distance between the parallel planes. Then, since, according to previous demonstration, the p. f. applies to ABCA′B′C′,

$$dV = \frac{h}{6}[dE + 4dM + dE']. \qquad (15)$$

The revolution of the warped surface, ABB′A′, generates E,M,E′,V, simultaneously, and completes the generation of all in the same time. Therefore, if an independent line be considered to be generated by the uniform motion of a point in the same time, and x be employed to denote its length at any time, the magnitude of upper base at such time, however irregular the generation be, may be expressed as $F(x)$, according to the demonstration of the introductory part of this paper. Similarly, M,E′,V, at any state are $F'(x), F''(x), F'''(x)$. Now, since the variables of eq. (15) are all functions of a single variable, x, if the integration of all the terms be performed between the limits $x=o$, $x=l$, or, when the independent line is the circumference of a circle, whose radius is unity, between the limits $x=o$, $x=2\pi$, there is obtained

$$V = \frac{h}{6}[E + 4M + E']; \qquad (16)$$

that is, the p. f. applies to the space.

If the integrals be confined by closer limits, it results that the p. f. applies to such a space as ABB′I′IAA′.

When the generating warped surface reaches the position AGG′A′, the next dE' is negative, while the corresponding dE is positive, because HH′ crosses the opposite face of the shape AGHH′G′A′A. Further on, both become negative, and remain so until one edge of differential solid again crosses its opposite face.

The limits of the integrals may be extended, such corresponding to the case of the revolution of AB through more than 360°. The unreal shape thus produced, may have parts of double, triple or greater intensities; also parts may be negative or null. The p. f. still applies.

The class of shapes which are bounded laterally by ruled surfaces, are very important for two reasons: *First*, nearly all ordinary prismoidal shapes are included. *Second*, when it is necessary to determine the volume of a space covered by a given unknown surface, it is possible to approximate it, readily and as nearly as is desirable, by means of ruled surfaces; whereas it is impossible by aid of the eye to approximate it by means of surfaces all of whose elements are given curves, because the eye cannot recognize the nature of a flat curve, but can always recognize a straight line. The plane, a

surface composed in every direction of straight lines, is not pliable enough to enable us to use it with accuracy for any but small extents of surface.

It is evident that this class of shapes may have very grotesque forms. For instance, at one place a logarithmic curve may extend from base to base, at another, the sinusoid, at another, a line represented by an algebraic equation of the hundredth degree, and at still another place may be a broken line; yet the generatrix can be moved so as to pass through every point of all.

The following somewhat similar rules are interesting, though they do not include many practical shapes.

The prismoidal formula applies to every shape bounded, terminally, by two parallel planes, and, laterally, by the surface generated by the motion of the ordinary parabola, returning, finally, to its initial position, but, meanwhile, undergoing any series of changes in amount and direction of curvature, position of principal vertex, and direction of containing plane, the principal axis, however, always remaining parallel to any line in either base.

The prismoidal formula applies to any shape contained by two parallel bases, and a lateral surface generated by the motion of a cubic parabola whose plane is always parallel to a given plane, but whose curvature and point of inflexion may pass through any series of changes in amount, direction and position.

It is often necessary to calculate the contents of ungulæ and their frusta. Sometimes the p. f. is applied to these. Let us examine this case in a general manner. Consider the shape whose surface is formed by revolving the line

$$y = a + bx + cx^2 + dx^3 + \text{etc.}$$

about the axis of X. Consider, also, a secant surface composed of straight line, parallel elements, perpendicular to plane XY. The equation of this surface is

$$y' = a' + b'x + c'x^2 + d'x^3 + \text{etc.}$$

At a distance x from origin, the magnitude of generatrix of either segment of the space, is a semicircle whose radius is y, plus or minus a segment of the same circle, included between diameter and chord distant from center by y'. Its area is, consequently,

$$\tfrac{1}{2}\pi y^2 \pm y'\sqrt{y^2 - y'^2} \pm y^2 \sin^{-1}\frac{y'}{y}.$$

In order that p. f. shall apply, this must be equivalent to an algebraic expression of third or less degree. The transcendental factor of the last term must, therefore, be constant. Hence, the proportion,

$$a' : a :: b' : b :: c' : c :: \text{etc.},$$

and $$y' : y :: A : 1 :: a' : a :: \text{etc.},$$

in which A is any constant quantity, must subsist. The second term now becomes $Ay^2\sqrt{1-A^2}$. From this it is seen that A must be less than unity, and that all coefficients beyond b, b', must be zero.

The interpretation of this is: The shape must be a cylinder, a cone or a frustum of a cone. The secant surface must be a plane, parallel to elements of cylinder, or passing through vertex of cone, or through vertex of the cone whose frustum is considered. On p. 34 of Trautwine's *Civil Engineer's Pocket-Book*, that author describes, and illustrates by diagram, a segment of a cylinder made by a secant plane not parallel to elements, but which is merely required to pass through both bases; and he denominates it as a space to which the p. f. applies exactly. The foregoing demonstration shows that this is not so. It is well to point this out, because an error becomes very important, when contained in a book otherwise so admirable as to claim the full confidence of engineers.

The magnitude of the generatrix of any space, is, at any position, a function of the corresponding ordinates of all the longitudinal elements of its boundary. Therefore, any number of these elements may be arbitrarily fixed, while enough remain free to have such values accorded their ordinates as shall render the value of the function, that is, the area of generatrix, whatever we please in terms of the path; and, as these remaining elements are infinite in number, there are an infinite number of solutions to this problem. This enables us to impose uncommon conditions upon prismoidal space, and, by selecting one of the solutions, to obtain the equation of an extraordinary prismoidal shape.

Thus, let it be required to determine the equation of the boundary of a prismoidal space, such that the line of double curvature,

$$\left\{\begin{array}{l} z=a+bx+cx^3-dx^5+ex^8 \\ y=g+hx^3-x^4+kx^7 \end{array}\right.$$

shall be contained in that boundary.

Let every cross section, parallel to the plane YZ, be an ellipse with its principal axes parallel to the axes of Y and Z. At a distance x, the equation of the cross section is

$$A^2(z-n)^2+B^2(y-m)^2=A^2B^2.$$

Of the four conditions which may be imposed upon this cross section, we exhaust one in making its perimeter pass through the given line, by assigning to z and y their values in the equations of that line. For the other conditions let

$$m=g+hx^3-x^4+kx^7,$$
$$n=-dx^5+ex^8,$$

and $A=a$ *constant*,

whence, $B=a+bx+cx^3$,

in order that the area, πAB, shall be

$$\pi A(a+bx+cx^3).$$

The last expression shows that p. f. applies to the space in question. Substituting in the general equation of the ellipse the values of A,B, m,n, in terms of x, and making x general, we obtain the equation of the shape,

$$\left.\begin{array}{l} A^2(z+dx^5-ex^8)^2 \\ +(a+bx+cx^3)^2[(y-g-hx^3 \\ \qquad +x^4-Kx^7)^2-A^2] \end{array}\right\}=o.$$

Let it be required to find the equation of a surface, symmetrical about the axis of Z, which shall contain the lines

$$x=\log.\ (a+z),$$
$$y=b+c\ \sin.\ z,$$

and shall be the boundary of a prismoidal space.

Let the equation of any cross section, parallel to XY, be

$$y^2=[g\sqrt{h^2-x^2}-k(h^2-x^2)]^2.$$

Assigning to x and y appropriate values, we find

$$h=\log.\ (a+z),$$
$$gh-kh^2-b+c\ \sin.\ z.$$

$$\int_o^h y\,dx=\tfrac{1}{2}g\left\{x\sqrt{h^2-x^2}+h^2\ \sin.^{-1}\frac{x}{h}\right\}_o^h$$

$$-k\left\{h^2x-\tfrac{1}{3}x^3\right\}_o^h$$

$$=\tfrac{1}{4}\pi gh^2-\tfrac{2}{3}kh^3=$$

$$\tfrac{1}{4}\pi\log\ (a+z)[b+c\ \sin z+k\ \log^2(a+z)]$$
$$-\tfrac{2}{3}k\ \log^3\ (a+z)$$
$$=m+nz+pz^2+qz^3.$$

The last member expresses the condition that the p. f. shall apply. m,n,p,q, are arbitrary. Assign values to these; solve equation with respect to k; knowing k and h determine g; then, substitute for g, h, k, their values in terms of z, in general equation of cross section, and make z general. The result is the equation of the required surface.

Consider the space bounded by the surfaces,

$$y=o,\ y=w,\ z=o,$$
$$z=ax^my^n+bx^ty^q+cx^ry^s+dx^ty^u+\text{etc.} \quad (17)$$

The exponents m,n,p, etc., of equation of top surface, having any given values, it is required to find the relation which must exist among the values of coefficients of this equation, in order that the p. f. shall apply in the direction of X.

Eq. (17) may be placed in this form.

$$z=\left\{\begin{array}{l} a_1+b_1x+c_1x^2+d_1x^3+\varepsilon_1x^4 \\ \qquad +g_1x^5+\text{etc.} \\ +(a_2+b_2x+c_2x^3+d_2x^3+\varepsilon_2x^4 \\ \qquad +g_2x^5+\text{etc.})y \\ +(a_3+b_3x+c_3x^3+d_3x^3+\varepsilon_3x^4 \\ \qquad +g_3x^5+\text{etc.})y^2 \\ +\text{etc.} \end{array}\right. \quad (18)$$

This last, for any value of x, is the equation of the corresponding transverse element of the surface. Therefore, the result of the multiplication of this general value by dy, and the integration between $y=o$, $y=w$, is the area of the generatrix at the distance x from origin. In order that the p. f. may apply between any limits, this area must equal

$$m+nx+px^2+qx^3, \quad (19)$$

where m, n, p, q, are arbitrary. Then

$$\left.\begin{array}{l} (a_1+b_1x+c_1x^2+d_1x^3+\varepsilon_1x^4+ \\ \qquad g_1x^5+\text{etc.})w \\ +\frac{1}{2}(a_2+b_2x+c_2x^2+d_2x^3+\varepsilon_2x^4 \\ \qquad +g_2x^5+\text{etc.})w^2 \\ +\frac{1}{3}(a_3+b_3x+c_3x^3+d_3x^3+\varepsilon_3x^4 \\ \qquad +g_3x^5+\text{etc.})w^3 \\ \qquad +\text{etc.} \\ -m-nx-px^2-qx^3 \end{array}\right\}=o, \quad (20)$$

is the criterion.

If we make the sum of all coefficients of the same power of x, equal to zero, eq. (20) is true. That is, eq. (20) is true, and the p. f. applies to shape covered by (17) or (18) when in latter one coefficient of a term containing x to each power, is such a function of w and all other coefficients of terms containing x to same power, as is implicitly expressed by the equation

$$g_1 w + \tfrac{1}{2} g_2 w^2 + \tfrac{1}{3} g_3 w^3 + \text{etc.} = o. \qquad (21)$$

Since m, n, p, q, are arbitrary, the coefficients of terms containing x to powers less than fourth, need not, if we please, be altered.

We may also, by alteration of the coefficient and exponent of x in any one chosen term of eq. (17), make this the equation of a surface covering prismoidal space. Thus, to one coefficient, as c_1, ε_2 or d_3, may be given such a value in terms of the variable x and all other constants in eq. (20), as to make that equation true.

Suppose eq. (17) contains two or more terms with x^4, and none with x to higher powers, whatever be the powers of y. If, now, in eq. (20), the coefficients, m, n, p, q, have all the sets of values possible, that is, if the expression of third degree, (19), have in succession all the values possible, in each instance one coefficient of x^4, or some other coefficient, may have such a value, according to eq. (21), as to make eq. (20) true. Therefore, there may be as many surfaces of the fourth degree, with respect to x, such that the p. f. applies to the space covered by it, as there are surfaces, altogether, susceptible of representation by a cubic equation. For every additional term in eq. (17) containing x^4, the number of such surfaces becomes infinitely greater. In general, the number of surfaces of any degree, covering prismoidal space, is, at least, equal to the whole number of surfaces of degree next lower, and it may be infinitely greater.

For a particular set of values of m, n, p, q, eq. (20) reduces to the more convenient form

$$\left.\begin{array}{l}(\varepsilon_1 x^4 + g_1 x^5 + \text{etc.},)w \\ + \frac{1}{2}(\varepsilon_2 x^4 + g_2 x^5 + \text{etc.},)w^2 \\ + \frac{1}{3}(\varepsilon_3 x^4 + g_3 x^5 + \text{etc.},)w^3 \\ \qquad + \text{etc.}\end{array}\right\} = o, \qquad (22)$$

which still represents an infinite number of conditions.

Any surface, which varies with x and y but not with z, as

$$y = \alpha + \beta x + \gamma x^2 + \Delta x^3 + \text{etc.}, \qquad (23)$$

may be substituted for $y = w$; and the variable value of y substituted for the constant value w, in expression (20), or (22), whereafter it will be necessary only to make one constant, as a_1, ε_2, etc., or α, β, etc., a certain function of the remaining quantities, that there may result the equations of the two surfaces, conditioned to enclose a prismoidal space.

It follows from the above that through any number of points, arbitrarily fixed, except that more than one be not in the same perpendicular from base, a surface may be passed, while, also, the width of shape may vary according to any given function of x, so that between any limits the p. f. shall apply to the space enclosed.

Such a surface can be passed through the points.

$$\begin{array}{l} x=1,\ y=4,\ z=4, \\ x=2,\ y=4,\ z=2, \\ x=3,\ y=4,\ z=3, \\ x=5,\ y=4,\ z=2, \\ x=3,\ y=1,\ z=5, \\ x=6,\ y=2,\ z=1, \end{array}$$

and the curve

$$y = x + \tfrac{1}{2},\ z = 2 \text{ Nap. log. } (1+y)$$

Thus, through first four points and the curve, at a point in same vertical plane with these, pass a line of fourth degree. Through the last two points, and the two points in same vertical plane of the two curves, pass a cubic parabola. With the three curves as directrices, and the plane ZY as a plane director, generate a surface by motion of a cubic parabola. The area of generatrix, at any distance, x, is now represented by an expression containing one arbitrary quantity. Assign such a value to this as shall make the expression equivalent to a cubic equation.

Between given limits in directions of X and Y, a surface of any degree may be found, such that to the space covered by which the p. f. shall apply in both directions. This can be shown most briefly by an example. Consider the space

$$z = ax^5y + bx^4y^3 + cx^3y^4 + dx^2y^5 + exy^2 + fx + g.$$

Integrate with respect to y, between

limits o and w, all terms containing x to higher powers than the third, and set result equal to zero, using as criterion (22).

$$o=\tfrac{1}{2}ax^3w^3+\tfrac{1}{4}bx^4w^4. \quad \therefore b=-\frac{2ax}{w^2}.$$

Substitute in original equation; then proceeding in like manner for y, l taking the place of w, we obtain

$$o=\tfrac{1}{3}cy^4l^3+\tfrac{1}{4}dy^4l^4. \quad \therefore d=-\frac{4c}{3l}.$$

The required equation is

$$z=ax^3y-\frac{2ax^3y^3}{w^3}+cx^3y^4-\frac{4cx^3y^4}{3l}+exy^3+fx+g.$$

In exactly same manner space may be treated, when its boundaries are represented by equations containing polar co-ordinates, r,θ,z. Now the area of generatrix is $\frac{1}{2}\int_o^{2\pi} r^3\,d\theta$, instead of $\int_o^w zdy$. Examples of prismoidal shapes are the following two:

$$r=ax^3\theta^3-\frac{7ax^3\theta^4}{40\pi^2}+c,$$

$$r=\sqrt{ax^4\theta^3-\tfrac{1}{3}\pi ax^4\theta+bx^3\theta+cx^2\theta^5+dx+g}.$$

The equation of every shape, or of every surface, is

$$o=\mathrm{A}r^m\theta^nz^p+\mathrm{B}r^q\theta^sz^t+\text{etc. ad inf.}$$

It may be shown that this space can be made subject to the p. f. by alteration of any one term of its equation.

No equation of finite degree, representing a bounding surface, can define the limits of applicability of the p. f., because surfaces of higher degrees enclose prismoidal spaces. It requires two equations of infinite degree, one the general equation of the surface, and the other the equation of condition, to define the limits. The number of equations of finite degree necessary is infinite.

When the bounding surface of prismoidal space is of high degree, there may be a correspondingly great number of negative parts in the complete space; but the generatrix, being considered as a whole, can change its sign no oftener than three times, since it varies as the ordinate of a cubic parabola.

The formula

$$v=\frac{l}{8}\left\{\mathrm{A}+3\mathrm{B}+3\mathrm{B}'+\mathrm{A}'\right\}, \qquad (24)$$

where the capital letters in parenthesis represent areas of cross sections, dividing length of space into equi-parts, has no greater extent of exact application than the ordinary p. f., though it depends upon four cross sections. So, the formula for rectangle,

$$v=al, \qquad (25)$$

where a is area of mid section, has equal extent of application with the trapezoidal formula, where $a=\frac{1}{2}[\mathrm{A}+\mathrm{A}']$. The same is noticeable in formulæ of wider application.

If we multiply the generatrix of space, $\mathrm{F}(x)$, by dx, and integrate between $-\frac{1}{2}l$, $+\frac{1}{2}l$, we obtain

$$v=l[a+\tfrac{1}{12}cl^2+\tfrac{1}{80}el^4+\text{etc.}]. \qquad (26)$$

This does not depend upon odd powers of x in F (x). For rectangle, (26) reduces to (25); and, since b does not appear, this is true for any trapezoid.

When F (x) is of second degree, another pair of cross sections must be introduced. Then,

$$\mathrm{A}=a-\tfrac{1}{2}bl+\tfrac{1}{4}cl^2,$$
$$\mathrm{A}'=a+\tfrac{1}{2}bl+\tfrac{1}{4}cl^2,$$
$$\mathrm{A}+\mathrm{A}'=2a+\tfrac{1}{2}cl^2,$$
$$\tfrac{1}{12}cl^2=\tfrac{1}{6}[\mathrm{A}-2a+\mathrm{A}'].$$
$$\therefore v=\frac{l}{6}[\mathrm{A}+4a+\mathrm{A}'];$$

and this is applicable to spaces of third degree.

If F (x) be of fourth degree, then,

$$\mathrm{A}+\mathrm{A}'=2a+\tfrac{1}{2}cl^2+\tfrac{1}{8}el^4,$$
$$\mathrm{B}+\mathrm{B}'=2a+\tfrac{1}{8}cl^2+\tfrac{1}{128}el^4.$$

Find from these $\frac{1}{12}cl^2$ and $\frac{1}{80}el^4$. Then, (26) is

$$v=\frac{l}{90}[7\mathrm{A}+32\mathrm{B}+12a+32\mathrm{B}'+7\mathrm{A}']. \qquad (27)$$

This is exact when F (x) is of fifth degree.

An interesting but lengthy method of forming these rules is found in Israel Lyon's *Treatise of Fluxions*, London, 1758, pp. 173–8. A very elegant method by Chauncey Wright, is published in the *Mathematical Monthly*, Mass., Oct. and Nov., 1858. Another method is shown in *Boole's Calculus of Finite Differences.*

Numerous writers have furnished theorems of more or less generality, defining whole classes of shapes subject to

the p. f. Want of space compels us to mention the names of a few only, and to omit their rules altogether. These chiefly relate to the shape shown in Fig. 6, and to the shapes formed by revolution of conic sections.

Thomas Simpson in 1750 decomposed the ordinary prismoid into two wedges, and derived the formula

$$v=\tfrac{1}{6}h[\mathrm{WL}+wl+(\mathrm{W}+w)(\mathrm{L}+l)].$$

The ordinary rule follows directly from this. The same mathematician devised the popular rule for mechanical quadratum, which bears his name. He could not have failed to notice the similarity between them.

Charles Hutton in his *Mensuration*, 1770, constructs the formula in its common shape. He demonstrates that it is applicable to all solids generated by the revolution of a conic section, and to their frusta.

Professor Gillespie of Union College demonstrated, in 1857, before the *American Association for the Advancement of Sciences*, that the formula is applicable to the space covered by the hyperbolic-paraboloid. This was a most important extension of its uses.

John Warner made similar demonstrations in his treatise on earthwork calculations, 1861.

Charles Baillairgé, a Canadian, proposed in a treatise on mensuration, 1866, to use the p. f. as an universal rule, for practical purposes, in quadrature and cubature. He mentions no new shapes, to which the formula applies exactly; but simply uses a succession of prismoids to approximate any given space, [Simpson's method], and indicates what shapes need a close assemblage of measured cross-sections, and those whose contents are exactly, or very nearly, represented by the p. f. in one application. His work is designed for artisans unskilled in mathematics, and is, undoubtedly useful in that respect.

Professor Rankine, the Rev. Mr. Billion of Montreal, and Professor R. Steckel of Alsace, France, have given excellent general rules. Those of last two are published in Mr. Baillairge's *Stereometrical Tableau.*

None of the foregoing writers produced widely general rules.

Chauncey Wright, in 1858, in Oct. number of the *Mathematical Monthly*, Cambridge, Mass., obtained, by a brief and elegant demonstration, the cubic equation, (11), as expressing the law of variation in magnitude of the generatrix of prismoidal spaces. This was the solution of the problem which had, probably, engaged the attention of many mathematicians before.

Prof. E. W. Hyde, of the University of Cincinnati, in 1876, published in the July number of the *Analyst*, Des Moines, Iowa, an article entitled, *Limits of the Prismoidal Formula.* He attempted to determine the limits of the formula's application by means of eqs. representing shapes, or the bounding surfaces. If in eq. (20) of present paper, m, n, p, q, be assigned such values as reduce that equation to the form of eq. (22), and then, to satisfy this, each remaining coefficient as g_1, e_2, etc., be made zero, the resulting class of shapes is that which he discusses. Evidently, the longitudinal elements can only be lines of third degree or less. Using the co-ordinates, x, θ, ρ, he shows that the p. f. applies to the shape,

$$\rho=\sqrt{x^3f(\theta)+x^2f_1(\theta)+xf_2(\theta)+f_3(\theta)},$$

where θ may have any exponent, but x must not be of higher degree than third. This paper is very interesting, and the formulæ include many, though not all, practical shapes.

The present writer was led to investigate the subject, by a desire to compare the range of a new and general center of gravity formula with that of the prismoidal formula. From special trials upon spheroids, paraboloids and other common shapes, they seemed to occupy the same field. The result and the formula itself shall be published shortly. Other than this, the most important result of this paper, eq (11), has been anticipated by Chauncey Wright. The writer has endeavored to make this result, if possible, more general, by commencing with the conception of every space, and by discussing equation (10) thoroughly.

The writer is especially indebted to Prof. E. W. Hyde for information and references.

A VISIT TO A MINING DISTRICT IN CHINA.

By G. J. MORRISON, M. Inst. C.E.

From "The Builder."

In China,—that great land of undeveloped wealth,—there are villages and whole districts where all the men one sees are covered, I might almost say clothed, with coal dust, for they wear little else in summer, and where, as there are no railways, one meets long trains of coal-laden mules, asses or camels, according to the nature of the country.

About ten or twelve miles to the west of Pekin, runs a river called the Hun Ho. This is a shallow river, fordable in many places in dry weather, but after heavy rains it is subject to great floods, which make its passage difficult and dangerous even by a ferry-boat. There is, however, one bridge across it which is always available.

Pekin is situated on an immense plain, but immediately to the west of the Hun Ho the country becomes hilly. Mount Conolly, as it is called by foreigners, which rises to a height of 5,000 feet or 6,000 feet above the sea, is the highest peak in the immediate neighborhood. The whole of these hills abound in coal, and the district is generally termed by foreigners the Chai Taeng district, from the name of a village about forty miles from Pekin, and near which some of the best coal-mines are situated. Being in the neighborhood in August, 1877, I had an opportunity of forming one of a party to visit these hills. The party consisted of Mr. Nicolson, of the Diplomatic service; Mr. Brenan, of the Consular service (who is thoroughly acquainted with the colloquial language) and myself.

Having started early one Monday morning, we crossed the Hun Ho by a ferry at a village called Ma Yu, and at once began to ascend the mountain-road leading to the mines. By ten o'clock we reached the entrance to the Ta Yao, or great mine, situated about eighteen miles west of Pekin, and 1,050 feet above the sea.

The mines here are not worked as most mines are in England, by shafts sunk vertically, but by "adits" or entrances from the side of the hill, as is sometimes done at home. The people look for some place where the seam of coal crops out at the surface. They then begin making a tunnel about four feet wide, and four feet six inches high, carefully lining the side and roof with timber, so as to prevent any of the earth falling in, and they work on in this way until they have arrived at good solid coal, with firm rock above and below, the portion near the surface being generally more or less loose and broken up. When they are fairly in the solid coal, they begin working out coal to the right and the left, as well as in front. If they were to work out a large area without taking any precautions, the roof would fall in; they therefore place temporary props of timber where they are working, and, in addition, they carry forward their main road by carefully continuing the timber work of the tunnel, and also on each side, outside the timber, building a wall of the rubbish which is excavated with the coal, so that, even if the roof of any portion of the working falls, they are sure of having a road through which they can reach the solid coal further in. The coal when originally deposited, was no doubt level, but since that date there have been great upheavals of the strata, and much of the coal now lies at a very steep angle, sometimes as much as sixty degrees to the horizontal. The road or tunnel, therefore, following as it does the seam of coal, is sometimes flat and sometimes steep, and as various local causes make it convenient, during the progress of the work, to carry the road to one side or the other, it happens that the main road in an old mine is as crooked as a corkscrew. There is yet one other peculiarity. Although the tunnel is begun so large that a man can enter by stooping, it often happens that, when the seam is thin and the roof and floor composed of hard rock, the miners grudge the expense of cutting it, and the road, therefore, gets lower and lower until the final limit is reached of a road so low that a man can only just crawl through it. Now, having

come to see the mines, I was determined to enter into one and see the workings for myself. We, therefore, addressed ourselves to some miners whom we found at the mouth of the Ta Yao. At first they thought it was all a joke, and it was a considerable time before they could be persuaded that I was in earnest. Then they said it was impossible; that one foreigner had tried some years previously and had failed; that I would ruin my clothes, and made various other objections, but finally we prevailed. And so, divesting myself of most of my clothing, I took a lamp and prepared to follow my guide to the depths of the mine. The lamp was a curiosity in its way. In principle, it was the same as the ordinary open lamp used in the Scottish mines; but in general appearance it resembled a teapot without a handle. It was carried by a string passed round the head; and, being globular, it rolled about a great deal, and as it must have weighed over 1¼ lbs., it was a serious addition to the troubles of the journey.

On entering, I found that all along the floors there was laid a ladder about two feet wide, having good large rungs eight inches or nine inches apart. The ladder was roughly made, but was tolerably level on the upper surface. My guide, who at first took a fatherly interest in me, at once signed to me to walk on the rungs of the ladder, and not to let my foot slip between, as, in the event of there being a hole, I might break my leg. Very soon the road got so steep that I had to sit on the ladder and let myself down gently, catching the rungs with my heels; then it became so steep that I had to turn my face to the ladder and descend in the ordinary way. For about 600 yards the road was of this varied description, but rather steep on the whole. I estimated the total fall in this distance to be about 500 feet. This part of the journey was tolerably easy. The roof was seldom less than three feet or three feet six inches high, so that the only difficulty consisted in passing the coal drawers, to whom I shall refer presently. After the first 600 yards, however, the affair changed. The road twisted a good deal, sometimes was uphill, sometimes down, and sometimes level, and the roof was very low. The total length of this portion was 1,000 or 1,200 yards; and the total fall must have been between 100 feet and 200 feet. Along almost the whole of this it was necessary to go on hands and knees or hands and feet; but this was not the worst. In several places the roof was so low that I could not go on hands or knees. I am, unfortunately, a trifle larger than an ordinary Chinese miner, and not being acquainted with the place, I probably required one or two inches more space than I should otherwise have done, for it must be remembered that the roof is not smooth, and if one thinks he can crawl under a projecting stone, and is mistaken, he will be painfully reminded of his error by the loss of a large piece of skin from his back, if he is not more seriously hurt. There was but one way for it, and that was to lie on my face, stretch out my arms, and, resting my weight on my hands and elbows, drag myself forward. There were two or three places where I had to do this for a length of twenty yards at a time, and I was very glad when I arrived at the working face, a distance of about a mile from the entrance, and still more glad when I again reached the fresh air, after having spent about three hours below ground. The coal which is here being worked, is a description of anthracite, but not so hard and clean as some found on the other side of the hill. The seam in this mine is only one foot nine inches to two feet thick.

It is extremely difficult to give a fair description of the method of working. Being ignorant of the language, I could ask no questions, and as all old workings are full of rubbish, it is impossible to tell with certainty what was done last, or what is about to be done next. The main road running from the surface was crossed at certain intervals by roads running at right angles. The miner ascends from the road to the working face, and cuts away some coal, which he rolls down hill to the road, whence it is taken away by the drawers. While this is going on he is obliged to put temporary props to support the roof, and after a certain time these become so numerous as to interfere materially with the coal reaching the road, and for this reason the cross roads are placed at the distance which is found by experience to be most convenient. While the miner is working between the

cross roads, the main road is carried lower, and a new road is opened, and another miner is set to work there. It must not, of course, be understood that all this goes on with regularity. There are faults and other difficulties to encounter. Sometimes the coal is too thin to pay, and in an old mine the workings are most irregular; but, judging from what I saw, this is the arrangement aimed at. At the working faces only the coal is cut away, neither floor nor roof being touched; the space in which the miner works is therefore only one foot nine inches high. It is hard to wriggle about in such a place, and the difficulty of working is very great. It is, therefore, not surprising that the output of the mine does not much exceed two piculs for each miner per day. A picul ought to contain 100 catties of $1\frac{1}{3}$ lbs. each; but the wretched practice exists in China of having piculs of 120, 140, or even 200 catties, just as at home we have stones of different weights. I imagine the output in this case was about 3 cwt. per miner.

The tools used by the miner are chisels about twelve inches long, and one-half inch thick, and a small hammer. With these he cuts holes in the coal, and afterwards breaks off lumps of considerable size by means of wedges. The pick is very little used in these mines, although it is used in the bituminous coal mines a few miles distant. The coal is drawn to the surface in baskets fixed to sledges. Each basket contains about 100 lbs. A man or a boy draws the basket by a leather strap, which passes along his back over one shoulder, and back again between his legs to the basket. He progresses on hands and feet, and draws the load after him, the sledge passing smoothly enough over the ladder, whether the ground is levelled or inclined. When the ladder is steep the process looks dangerous, as, if the man were to slip, or anything to give way, the man behind would almost certainly be killed. The maximum work of a man is four journeys per day. In returning to the surface where the road is steep, the drawer sits on his basket and slides down the ladder, using his heels to guide himself, and to regulate the speed.

The mines here are never troubled with water. When any appears, as it does occasionally, the miners stop work for a few days, and it runs away through the fissures in the strata. The ventilation is effected through openings to the surface at various places, which I understood were either old entrances, or places where the workings had come to the surface. It was stated distinctly that in this mine no special air-courses were cut. On the day of my visit the temperature of the mine was much lower than that of the external air, and natural ventilation would be sufficient to keep the mine fresh; but they told us they sometimes used fans below ground—"something like winnowing-machines," about four feet or five feet diameter.

They said that with thirty miners and thirty drawers the output of the mine was about sixty piculs, which I take to be about four tons and a half. The wages of the men are three tias (about 1s. 1d.) per day, and of boys two tias (about 9d.). This coal sells at the mine at three tias and a half a picul. This would appear to be a losing game, but the profit is made on the difference between the miners' picul and the selling picul. The mines are worked for the benefit of the owners of the land, and we were told that they paid nothing to Government, except a yearly present of small amount to the local magistrate.

After hearing this, I fancy most people will be surprised to find that the Chinese know so much and effect so little. The system of working is not unscientific, and only differs in detail from our long wall system at home. Although we place our fans for the ventilation of collieries at the surface, still, in the construction of headings, &c., they are often placed below. The Chinese would probably be unable to cope with large quantities of water; but it must not be forgotton that within a century our own Cornish miners failed in this, and only succeeded through the assistance of outsiders, of whom Watt was the chief. But with all this knowledge, here is a mine which has been worked for 100 years, and where the workings extend to the distance of a mile, with a daily output of four or five tons.

The great want below as above ground is that of roads. I am tolerably safe in saying that there is no drawing-road in any coal-mine in Great Britain like the one I have described. There are, doubtless,

air-courses as small, but then men only pass through them occasionally. The worst roads are to be found in the Cornish mines, where, unfortunately, it is a common thing for the miners to take more than an hour to get from the surface to their working places, and the owners comfort themselves by thinking that in old workings nothing better can be expected, and by trying to believe that it is the miners' time and strength that are sacrificed, and not theirs.

Dreadful, however, as is the life of these poor Chinese miners, it must not be supposed that they appear to be discontented. Their wages are more than sufficient to provide them with plenty of good food and with houses to live in, and as they wear no clothes below ground, and very little in summer above ground, their tailors' bills cannot be serious matters. They are always civil and ready to chat and joke with anyone, and here, as over the greater part of China, a foreigner has nothing to fear from the common people, unless, either intentionally or accidentally, he offends them.

I believe I am correct in stating that women are never employed in Chinese mines. In this respect the Chinese are far in advance of the Japanese, and, for that matter, of the Sicilians and some other western nations, in whose mines much of the drawing is done by girls. I have seen dozens of naked Japanese men working in the same mine with women, whose clothing was just visible to the naked eye.

Immediately to the west of the mine just described there is a low range of hills, and after passing these the scenery is very fine. The numerous coal-mines occasionally mar the landscape; but some of the cliffs are magnificent. The roads are execrable, taking one up and down hills at almost impossible gradients. Mules and asses are the only beasts of burden that can use them, and wheeled traffic is impossible. Twenty miles a day, as the crow flies, is hard work.

The following day we reached Chai Tang, about forty miles west of Pekin, and we visited some of the bituminous coal-mines in the neighborhood. The seams which we saw are six feet or seven feet thick, and there are some much thicker. The mines appear to be worked on the pillar and stall system; that is to say, large pillars of coal are left to support the roof, and these are worked out when any portion of the mine is about to be abandoned. Here they construct artificial air-courses, and also employ "brattices," or partitions of timber or similar material, to divide the passages in two, so that the air may travel in along one division, and out along the other. When asked about fire-damp, they replied that they were troubled with it occasionally, and in some neighboring mines there had been explosions killing people; but they added, "these only occur when the air does not circulate."

It will thus be seen that they have not a great deal to learn in the theory of mining. The want of roads is not so much felt in these mines as in the thin anthracite seams. These seams are high, and the mines have not been worked so long. One draw can bring out about a ton a day, and one miner can get as much. The tools used are picks with one head, that is to say, in the form of an ┐, and not of a ┬. We were told that the price at the pit's mouth was 6s. a ton, but I believe it is often sold as low as 4s. The cost of carriage to Pekin is, however, 30s., so the coal is dear enough there.

About thirty-five miles west of Pekin we came upon a Roman Catholic village of 800 or 1,000 inhabitants, called Sang Yu. Not far from this village we came upon a mill for grinding wood and other materials to make joss-sticks. These sticks are about twice the length of an ordinary pencil, and rather thinner, and of a light brown color. They are placed in front of the images in the temples, and when lighted, smoulder slowly, giving off a slight and not unpleasant odor. The mill was driven by a horizontal water-wheel,—a description of turbine. The wheel was eight feet or ten feet in diameter, and the spokes were flat and very wide. The water issued from a spout, and impinging on the spokes, drove the wheel round. The whole available fall of the water was not utilized, but as the axle of the wheel formed the vertical axle of the mill, there was absolutely no gearing whatever, and nothing that required skilled labor to erect, or was likely to get out of repair; and as there was abundance of water-

power to be had, the arrangement was very creditable to the designer.

The next day we visited the Imperial tile manufactory at Lien li ku, about fifteen miles west of Pekin. Here all the yellow tiles and bricks required for Imperial buildings are made, as also large numbers of green, blue, and other colored tiles, for various ornamental purposes. The material used is a hard blue shale, nearly as hard as slate. This is allowed to lie in heaps for some time. It is then ground to powder by granite rollers, on a stone floor thirty or forty feet in diameter. The powder is then stored in heaps, and taken to the works as required. For ordinary work the powder is mixed with a proper proportion of water, and moulded into large bricks, which are laid out to dry for some hours, after which they are dealt with by the modellers. When bricks are to have a moulding on them, say, for coping a wall, the plan of the operation is as follows: Two pieces of wood, each cut to the shape of the moulding, are placed upright on a slab. The clay brick is placed between them, and two men run the mouldings roughly along with chisels. They then apply straight-edges to test the accuracy of their work, and finally rub the edges with moulds somewhat in the same way as plasterers make mouldings at home. The brick is then passed to a third man, who cuts any necessary holes in it, and to a fourth, who trims it off and repairs any defect. The more ornamental tiles and bricks, representing fabulous animals, &c., are first roughly moulded, and afterwards finished off with tools exactly similar to those used for modelling in clay in Europe. Some of this work has some pretension to artistic merit. All the bricks and tiles are baked in ovens, and then, after having the glaze put on, are baked a second time. All the work done at this manufactory appears to be first-rate, and the number of people employed when they are busy is about 500.

Much of the work we saw, particlarly the moulding of coping bricks, could no doubt be executed easily by machinery; but seeing that labor is so cheap, I doubt if any advantage could be gained by its introduction. As an example of the paradoxes to be seen in China, it is worthy of remark that these bricks, which deserve to rank with the highest class of artificial building materials in the world, can only reach the capital, distance fifteen miles, by being carried on camels or mules.

The Chinese manners and customs, and modes of thought, differ so entirely from our own, that it is extremely difficult to express any general opinion regarding them. The most contrary views are stated by people who agree perfectly on other subjects, and even the same person is sure to bestow encomiums upon them at one time, and to speak of them as little better than savages at another, in a way that is most perplexing. For instance, the money of the bulk of the inhabitants consists of copper coins, varying in value in different districts, but generally worth about $\frac{1}{2}\frac{1}{5}$ of a penny each. These coins, however, differ so much in size and quality that it takes more time to select and count 1,000 of them than to perform the same operation for 1,000 sovereigns. Is it wonderful, then, that one set of people are fond to talk of the admirable system of the Chinese, which enables the poorest person to get the advantage of a turn of the market in buying a single egg, while others can only look upon it as a system for wasting the time of hundreds of thousands of shrofts (or money-counters) who might be better employed, and for extorting from the poorer classes payments under the name of exchange to an amount which is absolutely appalling. Again, every particle of night-soil is collected and employed as manure on the fields. While one man can talk of nothing but the almost unbearable nuisance caused by the collection and distribution of the unsavory stuff, another can speak only of the science and industry displayed by the people in utilizing the valuable materials which civilized nations allow to run to waste. These differences may, to some extent, be explained by saying that one set of people are inclined to look at the ends aimed at, while another set look more at the means employed; but I fear it will be long before the views of writers on China cease to present contradictions which appear inexplicable.

On one point most people are agreed, and thrt is, that of all the defects of China, want of means of internal com-

munication is one of the greatest. They have a few bridges, some useful (like the bridge at Foochou, over the river Min, which is a work of considerable magnitude), and some ornamental (like the marble bridge near the Palace in Pekin, which is unquestionably a most elegant structure); but while a few people may talk of such works as evidences of enterprise and public spirit, most people can only consider them to be monuments of disgrace to a nation which knows what such things are, and which will not take the trouble to repair the old ones and build new ones!

Of roads, I may say, although I have been forced to employ the word occasionally, that they are non-existent. It is the fashion among authors at present, when they cannot find English words to express their meaning, and sometimes, I suspect, when they are not very clear as to the meaning they wish to express, to employ words from some other language. Any one describing the tracks through China, may be excused if he makes use of the Chinese name "Loo," for it is a fact, and one for which I am thankful, that it has no equivalent in English.

ELEMENTS OF THE MATHEMATICAL THEORY OF FLUID MOTION.

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IV.

§ 5.

FREE VORTEX MOTION.

We have already seen under what circumstances it is impossible for rotational motion to exist in a fluid mass. If the fluid in its initial condition has irrotational motion—or, if it be at rest, and motion is induced by a system of conservative forces, then the motion will always be irrotational; *i. e.*, if the quantity

$$udx+vdy+wdz$$

is at any time an exact differential it will always be one. The conditions for this quantity being an exact differential are

$$\frac{dw}{dy}-\frac{dv}{dz}=o,\ \frac{du}{dz}-\frac{dw}{dx}=o,\quad \&c.$$

Suppose that these quantities are not equal to zero, but that we have

$$\xi=\frac{1}{2}\left(\frac{dw}{dy}-\frac{dv}{dz}\right),$$
$$\eta=\frac{1}{2}\left(\frac{du}{dz}-\frac{dw}{dx}\right),$$
$$\zeta=\frac{1}{2}\left(\frac{dv}{dx}-\frac{du}{dy}\right);$$

The quantities ξ, η and ζ, as is well known, denote the components of angular velocity around the axes of x, y and z, respectively, of a particle whose velocities parallel to these axes are u, v, w. That these quantities should have certain definite values different from zero is, of course, the condition that vortex, or rotational motion exist in the liquid. These values of ξ, η and ζ, pre-suppose that we know the values of u, v and w. A problem that now immediately presents itself for solution is to find the values of u, v and w, supposing ξ, η and ζ to be given.

Assume three functions U, V and W such that

$$u=\frac{d\mathrm{W}}{dy}-\frac{d\mathrm{V}}{dz},$$
$$v=\frac{d\mathrm{U}}{dz}-\frac{d\mathrm{W}}{dx},$$
$$w=\frac{d\mathrm{V}}{dx}-\frac{d\mathrm{U}}{dy},$$

The quantities u, v and w must satisfy the equation

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=o.$$

It is found without difficulty that this equation will only be satisfied by the above values of u, v and w if the following conditions hold,

$$\frac{d^2U}{dx^2}+\frac{d^2U}{dy^2}+\frac{d^2U}{dz^2}=-2\xi,$$
$$\frac{d^2V}{dx^2}+\frac{d^2V}{dy^2}+\frac{d^2V}{dz^2}=-2\eta,$$
$$\frac{d^2W}{dx^2}+\frac{d^2W}{dy^2}+\frac{d^2W}{dz^2}=-2\zeta,$$
$$\frac{dU}{dx}+\frac{dV}{dy}+\frac{dW}{dz}=o.$$

The integrals of the first three of these equations are well known to be given by

$$U=\frac{1}{2\pi}\int\int\int\frac{\xi'dx'dy'dz'}{\sqrt{(x'-x)^2+(y'-y)^2+(z'-z)^2}},$$

$$V=\frac{1}{2\pi}\int\int\int\frac{\eta'dx'dy'dz'}{\sqrt{(x'-x)^2+(y'-y)^2+(z'-z)^2}},$$

$$W=\frac{1}{2\pi}\int\int\int\frac{\zeta'dx'dy'dz'}{\sqrt{(x'-x)^2+(y'-y)^2+(z-z)^2}};$$

where x',y,z' are the co-ordinates of any other point in the vortex element, and $\xi,'\ \eta,'\ \zeta'$ are the angular velocities at this point; the denominator,

$$r=\sqrt{(x'-x)^2+(y'-y)^2+(z'-z)^2}$$

denotes the distance between this point and the assumed point to which the U,V, W refer. Before going further it will be convenient to give two of Helmholtz's definitions. The line passing through any point and coinciding at all times in direction with the instantaneous axis of rotation of that point is called a *vortex line.* If we consider a number of vortex lines passing through every point in the perimeter of an infinitely small surface, they will cut from the rest of the fluid a filament which is called a *vortex filament,* or a vortex filament is an infinitely small filament of the fluid whose bounding surface is made up of vortex lines.

Now, in our equations giving U, V, W, the points x, y, z and x' y', z' are supposed to lie on the same vortex filament; we can represent an element of this filament by $d\tau$, then our equations become,

$$U=\frac{1}{2\pi}\int\frac{\xi'd\tau}{r},$$
$$V=\frac{1}{2\pi}\int\frac{\eta'd\tau}{r},$$
$$W=\frac{1}{2\pi}\int\frac{\zeta'd\tau}{r}.$$

where the integrations are of course extended over all the space which is supposed to be filled with vortex filaments. Now to examine the condition

$$\frac{dU}{dx}+\frac{dV}{dy}+\frac{dW}{dz}=o.$$

By differentiation we have

$$\frac{dU}{dx}=-\frac{1}{2\pi}\int\int\int\frac{(x-x')\,\xi'dx'dy'dz'}{r^3}$$
$$\frac{dV}{dy}=-\frac{1}{2\pi}\int\int\int\frac{(y-y')\,\eta'dx'dy'dz'}{r^3}$$
$$\frac{dW}{dz}=-\frac{1}{2\pi}\int\int\int\frac{(z-z')\,\zeta'dx'dy'dz'}{r^3}$$

Integrating by parts we have,

$$\frac{dU}{dx}=-\frac{1}{2\pi}\int\int\frac{\xi'dy'dz'}{r}+\frac{1}{2\pi}\int\int\int\frac{1}{r}\frac{d\xi'}{dx'}dx'dy'dz'$$

$$\frac{dV}{dy}=-\frac{1}{2\pi}\int\int\frac{\eta'dx'dz'}{r}+\frac{1}{2\pi}\int\int\int\frac{1}{r}\frac{d\eta'}{dy'}dx'dy'dz'$$

$$\frac{dW}{dz}=-\frac{1}{2\pi}\int\int\frac{\zeta'dx'\,dy'}{r}+\frac{1}{2\pi}\int\int\int\frac{1}{r}\frac{d\xi'}{dz'}\,dx'dy'dz'$$

Now since

$$\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz}=o$$

throughout the entire mass of the fluid we have

$$\frac{dU}{dx}+\frac{dV}{dy}+\frac{dW}{dz}=-\frac{1}{2\pi}\left\{\int\int\frac{\xi'dy'dz'}{r}+\int\int\frac{\eta'dx'dz'}{r}+\int\int\frac{\zeta'dx'dy'}{r}\right\}$$

This can readily be changed into a surface integral. If $d\sigma$ denote an element of the surface of the vortex filament and cos α, cos β, cos γ, denote the direction cosines of the outward normal to this surface, we have

$$dx'dy'=d\sigma\cos\gamma,\ dx'dz'=d\sigma\cos\beta,$$
$$dy'dz'=d\sigma\cos\alpha;$$

therefore our integral becomes

$$\frac{1}{2\pi}\int\frac{1}{\sigma}[\xi'\cos\alpha+\eta'\cos\beta+\zeta'\cos\gamma]d\sigma$$

taken over the entire surface. Now from the equation

$$\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz}=o$$

we have also

$$\int\int\int\left(\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz}\right)dx\,dy\,dz=o$$

by integration this becomes

$$\int\int\xi\,dy\,dz+\int\int\eta\,dx\,dz+\int\int\zeta\,dx\,dy=o$$

or as a surface integral,

$$\int(\xi\cos\alpha+\eta\cos\beta+\zeta\cos\gamma)d\sigma=o$$

consequently

$$\frac{d\mathrm{U}}{dx}+\frac{d\mathrm{V}}{dy}+\frac{d\mathrm{W}}{dz}=o.$$

Substituting now the obtained values of U,V,W, in the equations $u=\frac{d\mathrm{W}}{dy}-\frac{d\mathrm{V}}{dz}$ &c... and we obtain for u, v, w, the following values:

$$u=-\frac{1}{2\pi}\int\int\int\frac{1}{r'^2}[\zeta'(y-y')-\eta'(x-x')]dx'dy'dz'$$

$$v=-\frac{1}{2\pi}\int\int\int\frac{1}{r'^2}[\xi'(z-z')-\zeta'(x-x')]dx'dy'dz'$$

$$w=-\frac{1}{2\pi}\int\int\int\frac{1}{r'^2}[\eta'(x-x')-\xi'(y-y')]dx'dy'dz'$$

or, as these may be expressed,

$$u=\frac{1}{2\pi}\int\left(\zeta\frac{d\frac{1}{r}}{dy}-\eta\frac{d\frac{1}{r}}{dz}\right)d\tau$$

$$v=\frac{1}{2\pi}\int\left(\xi\frac{d\frac{1}{r}}{dz}-\zeta\frac{d\frac{1}{r}}{dx}\right)d\tau$$

$$w=\frac{1}{2\pi}\int\left(\eta\frac{d\frac{1}{r}}{dx}-\xi\frac{d\frac{1}{r}}{dy}\right)d\tau$$

Representing each of these differential expressions by u', v', w' respectively we see that u', v', w' are the increments of u, v, w, which correspond to the element $dx\,dy\,dz$ of the vortex filament. Writing for convenience of reference the equations

$$u'=\frac{d}{2\pi}\left(\zeta\frac{d\frac{1}{r}}{dy}-\eta\frac{d\frac{1}{r}}{dz}\right)$$

$$v'=\frac{d\tau}{2\pi}\left(\xi\frac{d\frac{1}{r}}{dz}-\zeta\frac{d\frac{1}{r}}{dx}\right)$$

$$w'=\frac{d\tau}{2\pi}\left(\eta\frac{d\frac{1}{r}}{dx}-\xi\frac{d\frac{1}{r}}{dy}\right)$$

we see that they give rise to the equation

$$\xi u'+\eta v'+\zeta w'=o.$$

This shows that, considering u', v', w' as the components of a certain new velocity, the direction of the resultant

$$\sqrt{u'^2+v'^2+w'^2}$$

of these components is at right angles angles to the direction of the axis of rotation of the element $d\tau$. Again, we have

$$u'\frac{dr}{dx}+v'\frac{dr}{dy}+w'\frac{dr}{dz}=o$$

and the direction of this resultant is also at right angles to the line r joining the element $d\tau$ to any other. Thus we see that each rotating element of the fluid mass implies in every other element a velocity whose direction is at right angles at the same time to the axis of rotation of the first and to the line joining the two elements—*i.e.*, at right angles to the plane containing the second element and the axis of rotation of the first. It is easily shown that

$$\sqrt{u'^2+v'^2+w'^2}=\frac{d\tau}{2\pi}\sqrt{\xi^2+\eta^2+\zeta^2}\,\frac{\sin\vartheta}{r^2}$$

when ϑ denotes the angle between r and the axis of rotation. From this equation we see that the magnitude of this induced velocity is directly proportional to the volume of the first element, its angular velocity and the sine of the angle between the line joining the two elements and the axis of rotation; and also inversely proportional to the square of the distance between the elements. Denote the angular velocities at the time $t=o$ by ξ_o, η_o, ζ_o then the last equations of chapter I become

$$\xi=\xi_o\frac{dx}{da}+\eta_o\frac{dx}{db}+\zeta_o\frac{dx}{dc}$$

$$\eta=\xi_o\frac{dy}{da}+\eta_o\frac{dy}{db}+\zeta_o\frac{dy}{dc}$$

$$\zeta=\xi_o\frac{dz}{da}+\eta_o\frac{dz}{db}+\zeta_o\frac{dz}{dc}$$

a, b, c being the co-ordinates of an arbitrary particle we have for the co-ordinates of another situated indefinitely near this $a+da$, $b+db$, $c+dc$, and at the time t the co-ordinates will be x, y, z, $x+dx$, $y+dy$, $z+dz$; now suppose that at $t=o$ we have

$$\frac{da}{\xi_o}=\frac{db}{\eta_o}=\frac{dc}{\zeta_o}$$

that is da, db, cd proportional to the initial angular velocities—and suppose further that the direction of this indefinitely small line coincides with that of the axis of rotation. Call the common value of these ratios ε, ε being an indefinitely small quantity, independent of the time; then we have

$$da=\xi_0\varepsilon,\ db=\eta_0\varepsilon,\ dc=\zeta^0\varepsilon,$$

Substituting these values in the above equations, and we have

$$da=\xi\varepsilon,\ dy=\eta\varepsilon,\ dz=\zeta z,$$

or

$$\frac{dx}{\xi}=\frac{dy}{\eta}=\frac{dz}{\zeta}$$

consequently the direction of the line joining the indefinitely near elements will at all times coincide with the direction of the axis of rotation. This combined with our definition of a vortex line shows us that every particle of fluid that lies on a vortex line at any instant will always remain there. If we call ω the resultant angular velocity we have

$$\omega=\sqrt{\xi^2+\eta^2+\zeta}=\frac{1}{\varepsilon}\sqrt{dx^2+dy^2+dz^2}$$

or the angular velocity so varies as to remain always proportional to the distance between the two particles. We have all along supposed the density of the fluid equal to unity. Remembering now our definition of a vortex filament, we see that any vortex filament must remain composed of the same fluid particles. Calling now k the cross section of any filament and l an indefinitely small length of the filament, that is

$$l=\sqrt{dx^2+dy^2+dz^2}$$

we have kl=const.; but l is proportional to ω, therefore ωk=const. or the product of the angular velocity of an infinitely small portion of any filament into its cross section is a constant. Call now k_1, k_2 the cross sections of a filament at points whose angular velocities are given by ω_1, ω_2; and let $d\tau$ denote an element of the filament;

$$\int d\tau\left(\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz}\right)$$
$$=-\int d\sigma[\xi\cos(nx)+\eta\cos(ny)$$
$$+\zeta\cos(nz)]=-\int d\sigma\omega\cos(\omega n).$$

Now from the values of ξ, η, ζ, we see that the factor of $d\tau$ in the left hand integral is=o;

$$\therefore\int d\sigma\omega\cos(\omega n)=o$$

But at the ends of the portion of the vortex filament that we are considering, we have $\cos(\omega n)=\pm 1$, and for all other points $\cos(\omega n)=o$, and consequently our integral is equivalent to

$$\omega_1k_1-\omega_2k_2=o$$

or the product of the angular velocity and the cross section is a constant throughout the vortex filament. As each rotating element of the fluids implies rotation in every other element, we have that all the particles of fluid must be in motion, and consequently from the definition of vortex lines we see that these lines and consequently the vortex filaments cannot terminate within the fluid, but must either terminate in its surface or must return into themselves; the former of these cases is illustrated by the vortices formed in running water, and the latter by smoke rings.

If in the expression

$$\int\int\int d\tau\frac{d(w\mathrm{V}-v\mathrm{W})}{dx}+\frac{d(u\mathrm{W}-w\mathrm{U}}{dy}$$
$$+\frac{d(v\mathrm{U}-u\mathrm{V})}{dz}$$

we substitute for u, v and w their values in terms of the derivatives of U,V,W, we find that this is equal to zero, and from it we obtain the striking equation

$$\int d\tau u\left(\frac{d\mathrm{W}}{dy}-\frac{d\mathrm{V}}{dz}\right)+v\left(\frac{d\mathrm{U}}{dz}-\frac{d\mathrm{W}}{dx}\right)$$
$$+w\left(\frac{d\mathrm{V}}{dx}-\frac{d\mathrm{U}}{dy}\right)$$
$$=\int d\tau\mathrm{U}\left(\frac{dw}{dy}-\frac{dv}{dz}\right)+\mathrm{V}\left(\frac{du}{dz}-\frac{dw}{dx}\right)$$
$$+\mathrm{W}\left(\frac{dv}{dx}-\frac{du}{dy}\right)$$

or

$$\int d\tau u^2+v^2+w^2=2\int\int\int d\tau$$
$$[\mathrm{U}\xi+\mathrm{V}\eta+\mathrm{W}\zeta]$$

But we may write for U, V, W the values

$$\mathrm{U}=\frac{1}{2\pi}\int\frac{\xi'd\tau'}{r}$$
$$\mathrm{V}=\frac{1}{2\pi}\int\frac{\eta'd\tau'}{r}$$

$$W=\frac{1}{2\pi}\int\frac{\zeta' d\tau'}{r}$$

$$\therefore d\tau\, u^2+v^2+w^2=\frac{1}{\pi}\int\frac{d\tau'}{r}[\xi'\xi'+\eta\eta'+\zeta\zeta']$$

But the expression for the energy of the fluid is

$$T=\tfrac{1}{2}\int d\tau(u^2+v^2+w^2)$$

and by virtue of the above we have

$$T=\frac{1}{2\pi}\int\int\frac{d\tau d\tau'}{r}[\xi\xi'+\eta\eta'+\zeta\zeta']$$

We can now take up the simplest case of vortex motion viz., that in which the motion is parallel to one plane. If we assume this plane as xy and further make the motion independent of z we have the angular velocities around x, and y, and the velocity in direction of z equal to zero, or

$$\xi=\eta=w=o$$

We have thus

$$u=\frac{dW}{dy},\quad v=-\frac{dW}{dx},\quad \zeta=\frac{dv}{dx}-\frac{du}{dy}$$

from which

$$-\zeta=\frac{d^2W}{dx^2}+\frac{d^2W}{dy^2}$$

which gives

$$W=-\frac{1}{\pi}\int\zeta\log\rho\, d\sigma$$

when $d\sigma$ is an element of the plane xy. Of course as ζ is independent of z we might have obtained this by integration from our general value of W before given. Here ρ represents the distance of the element $d\sigma$ in the plane xy from any other point in that plane. Each vortex filament implies in any other particle of the fluid a velocity whose components are,

$$-\frac{1}{\pi}\frac{\zeta d\sigma}{\rho}\cdot\frac{d\rho}{dy}\text{ and }\frac{1}{\pi}\cdot\frac{\zeta d\sigma}{\rho}\cdot\frac{d\rho}{dx}$$

and whose magnitude is

$$\frac{1}{\pi}\cdot\frac{\zeta d\sigma}{\rho}$$

The direction of this velocity is given by the cosines

$$-\frac{d\rho}{dy},\quad\frac{d\rho}{dx}$$

and of the line ρ by

$$\frac{d\rho}{dx},\quad\frac{d\rho}{dy}$$

or the direction of the velocity is at right angles to ρ. Assume two quantities x_0, y_0, which define by the equations

$$x_0\int\zeta d\sigma=\int x\zeta d\sigma,$$
$$y_0\int\zeta d\sigma=\int y\zeta d\sigma.$$

Now if we regard ζ as the density of a mass distributed over the element $d\sigma$ of the plane xy we see that x_0, y_0 will represent the co-ordinates of the center of gravity of this mass. Now l, the length of an indefinitely small portion of our vortex filament cannot alter, consequently by virtue of the equation kl=constant, k or $d\sigma$ cannot vary with the time and by virtue of $k\omega$=constant, ω or ζ cannot vary with respect to the time; consequently, we have, by differentiating with reference to t

$$\frac{dx_0}{dt}\int\zeta d\sigma=\int\frac{dx}{dt}\zeta d\sigma$$

$$\frac{dy_0}{dt}\int\zeta d\sigma=\int\frac{dy}{dt}\zeta d\sigma$$

Substituting

$$\frac{dx}{dt}=u=-\frac{1}{\pi}\int\frac{\zeta' d\sigma'}{\rho},\ \frac{y-y'}{\rho'}$$

$$\frac{dy}{dt}=v=\frac{1}{\pi}\int\frac{\zeta' d\sigma'}{\rho},\ \frac{x-x'}{\rho}$$

we have

$$\frac{dx_0}{dt}\int\zeta d\sigma=-\frac{1}{\pi}\int\int\rho\rho' d\sigma d\sigma'\frac{y-y'}{\rho^2}$$

$$\frac{dy^0}{dt}\int\zeta d\sigma=\frac{1}{\pi}\int\int\rho\rho' d\sigma d\sigma'\frac{x-x'}{\rho^2}$$

The double integrals are$=o$ therefore

$$\frac{dx_0}{dt}=o\quad\frac{dy_0}{dt}=o$$

or the center of gravity of the filament does not change with the time. In the case of only one vortex filament let us write

$$\int\zeta d\sigma=m$$

for particles as at finite distance from the filament we have

$$u=\frac{dW}{dy}\ v=-\frac{dW}{dx},\ W=-\frac{1}{\pi}\,m\log\rho$$

for particles indefinitely near the filament we see that W, u, v, are infinite and depend upon the cross section of the fila-

ment and the angular velocity ρ. We also know that at the center of gravity of the filament u and $v=o$. Each particle of fluid that is at a finite distance from the filament we see has a uniform velocity of $\frac{m}{\pi\rho}$ and moves in a circle whose center is the center of gravity of the vortex filament. Suppose we now assume a number of filaments whose cross section is indefinitely small. Write in general

$$\int_{\sigma_i} d\sigma_i = m_i$$

and let x_i y_i denote the co-ordinates of the centers of gravity of the filaments at the time t and ρ_i their distances from the point (xy). Then for all points at finite distances from the filaments we have as before

$$u=\frac{d\mathrm{W}}{dy},\ v=-\frac{d\mathrm{W}}{dx},\ \mathrm{W}=-\frac{1}{\pi}\Sigma m_i \log \rho_i$$

Now each filament inducing a certain amount of motion in every other particle of the fluid induces motion in the centers of gravity of every other filament, therefore the filaments change their places in the fluid. But here, as before, that portion of u, v which each vortex filament gives to its own center of gravity is $=o$. Suppose that the point from which the ρ_s are measured is one of the centers of gravity, for example x y. This will materially simplify the investigation by confining us exclusively to the influence of the system of vortices upon its different members and as this point x, y, is an arbitrary point no generality is lost. We have thus

$$u_1=\frac{d\mathrm{W}_1}{dy_1},\ v_1=-\frac{d\mathrm{W}_1}{dx_1}$$

$$\mathrm{W}_1=-=\frac{1}{\pi}$$

$$(m_2 \log \rho_{12}+m_3 \log \rho_{13}+\ldots m_i \log \rho_{1i})$$

or briefly

$$\mathrm{W}_1=-\frac{1}{\pi}\sum_{i=2}^{i} m_i \log \rho_{1i}.$$

We can now assume a function Q such that

$$\mathrm{Q}=-\frac{1}{\pi}\Sigma m_i m_j \log \rho_{ij}$$

and then we have

$$m_1\frac{dx_1}{dt}=\frac{d\mathrm{Q}}{dy_1},\ m_2\frac{dx_2}{dt}=\frac{d\mathrm{Q}}{dy_2},\ \ldots\ldots$$

$$m_1\frac{dy_1}{dt}=-\frac{d\mathrm{Q}}{dx_1},\ m_2\frac{dy_2}{dt}=-\frac{d\mathrm{Q}}{dy_2},\ \ldots\ldots$$

A complete system of integrals cannot be in general obtained, but by observing one peculiarity of Q we can obtain two integrals—

$$\rho_{ij}=\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}$$

If we increase x_i, x_j, or y_i, y_j, by the same quantity, ρ_{ij} will be unaltered; this gives us then

$$\Sigma\frac{d\mathrm{Q}}{dx_i}=o,\ \Sigma\frac{d\mathrm{Q}}{dy_i}=o$$

or

$$\Sigma m_i\frac{dy_i}{dt}=o,\ \Sigma m_i\frac{dx_i}{dt}=o$$

from which

$$\Sigma m_i x_i = \text{const.}$$

$$\Sigma m_i y_i = \text{const.}$$

or the center of gravity of the system of vortex filaments is unaltered. Again since

$$m_i\left(\frac{dx_1}{dt}dy_i-\frac{dy_i}{dt}dx_i\right)=o$$

we have

$$d\mathrm{Q}=o,\ \therefore\ \mathrm{Q}=\text{const.}$$

the equation of the lines of flow of the fluid. Introduce now polar co-ordinates,

$$x_1=\rho_1\cos\vartheta_1\quad x_2=\rho_2\cos\vartheta_2+\ldots.$$
$$y_1=\rho_1\sin\vartheta_1\quad y_2=\rho_2\sin\vartheta_2+\ldots.$$

we have by these substitutions

$$m_1\rho_1\frac{d\rho_1}{dt}=\frac{d\mathrm{Q}}{d\vartheta_1},\ m_2\rho_2\frac{d\rho_2}{dt}=\frac{d\mathrm{Q}}{d\vartheta_2}\ldots\ldots\ldots$$

$$m_1\rho_1\frac{d\vartheta_1}{dt}=-\frac{d\mathrm{Q}}{d\rho_1},\ m_2\rho_2\frac{d\vartheta_2}{dt}=-\frac{d\mathrm{Q}}{d\rho_2}\ldots$$

If now we increase all the ϑ's by the same quantity, Q will evidently remain unaltered and we have the equation,

$$\Sigma\frac{d\mathrm{Q}}{d\vartheta_i}=o$$

The first row now gives by addition

$$\Sigma m_i\frac{d\rho_i}{dt}=o$$

or

$$\Sigma m_i \rho_i{}^2=\text{const.}$$

Now let us suppose d to remain unchanged but ρ to become $n\rho$, then $\log\rho$ becomes $\log\rho+\log n$, and $\log\rho_{ij}$ becomes $\log\rho_{ij}\ 1+\log n$, and in consequence Q will be increased by

$$-\frac{1}{\pi}[m_1m_2\log n+m_1m_3\log n+m_2m_3\log n+\ldots.m_j\,n_j\log n+]$$

or

$$-\frac{1}{\pi}\log n\,\Sigma\,m_i\,m_j$$

and consequently we have

$$\Sigma\frac{d\,Q}{d\log\rho_i}=-\frac{1}{\pi}\Sigma\,m_i\,m_j$$

or

$$\Sigma\rho_i\frac{dQ}{d\rho_i}=-\frac{1}{\pi}\Sigma\,m_i\,m_j$$

But we have also

$$\frac{d\,Q}{d\rho_i}=-\,m_i\,\rho_i\,\frac{d\vartheta_i}{dt}$$

Substituting this value gives

$$\Sigma\,m_i\,\rho_i^2 d\vartheta_i=\frac{dt}{\pi}\,\Sigma m_i\,m_j$$

Assume now the case of only these vortex filaments existing in the fluid.

The equations $\Sigma m_i x_i$ = const.; and $\Sigma m_i y_i$ = const. become

$$m_1\rho_1\cos\vartheta_1+m_2\rho_2\cos\vartheta_2+m_3\rho_3\cos\vartheta_3=C_1$$
$$m_1\rho_1\sin\vartheta_1+m_2\rho_2\sin\vartheta_2+m_3\rho_3\sin\vartheta_3=C_2$$

Multiply the first equation by cos ϑ, the second by sin ϑ and add,

$$m_1\rho_1+m_2\rho_2\cos(\vartheta_2-\vartheta_1)+m_3\rho_3\cos(\vartheta_3-\vartheta)=C_1\cos\vartheta_1+C_2\sin\vartheta_1$$

Again multiply the first by sin ϑ, and the second by cos ϑ, and add

$$m_2\rho_2\sin(\vartheta_2-\vartheta_1)+m_3\rho_3\sin(\vartheta_3-\vartheta_1)=C_2\cos\rho_2-C_1\sin\vartheta_1$$

Again,

$$Q=-\frac{1}{\pi}[m_1m_2\log\rho_{12}+m_1m_3\log\rho_{13}+m_2m_3\log\rho_{23}+\ldots.]=\text{const.}$$

and

$$m_1\rho_1^2+m_2\rho_2^2+m_3\rho_3^2=\text{const.}$$

Through these four equations we may express any four of the quantities $\rho_1,\rho_2,\rho_3,\vartheta_2-\vartheta_1,\vartheta_3-\vartheta_1$ interim of the fifth; for example ρ_1; then the equations

$$m_1\rho_1 d\vartheta_1=-\frac{dQ}{d\rho_1}dt$$

and

$$\sum_1^3 m_i\rho_i^2 d\vartheta_i=\frac{dt}{\pi}\sum_1^3 m_i m_j$$

will enable us to express ϑ_1 and t as functions of ρ_1 and afford a complete solution of the problem.

Assume now only two vortices, and take the origin of co-ordinates at their common center of gravity. This point does not move, and we have

$$\frac{d\vartheta_1}{dt}=\frac{d\vartheta_2}{dt}$$

Q in this case becomes

$$-\frac{1}{\pi}m_1m_2\log(\rho_1+\rho_2)=\text{const.}$$

and also

$$\Sigma m_i\rho_i^2=m_i\rho_i^2+m_i\rho_i^2=\text{const.}$$

from these two equations we obtain

$$\rho_1=\text{const.},\quad \rho_2=\text{const.}$$

Again the equation

$$\Sigma m_i\rho_2^2 d\vartheta_1=\frac{dt}{\pi}\Sigma m_i m_j$$

becomes

$$m_1\rho_1^2 d\vartheta_1+m_2\rho_2^2 d\vartheta_2=\frac{dt}{\pi}\cdot m_1m_2$$

giving

$$\frac{d\vartheta_1}{dt}=\frac{d\vartheta_2}{dt}=\frac{1}{\pi}\cdot\frac{m_1m_2}{m_1\rho_1^2+m_2\rho_2^2}$$

If the direction of rotation of both vortex filaments is the same m_1m_2 [which depend on $\dot{c}_1$ and $\dot{c}_2$] have the same sign. But suppose $m_1=-m_2$, then

$$\frac{d\vartheta_1}{dt}=\frac{1}{\pi}\frac{m_1}{\rho_2^2-\rho_1^2}$$

But we have now for the center of gravity

$$x=\frac{m_1x_1-m_1x_2}{m_1-m_1}=\infty$$

$$y=\frac{m_1y_1-m_1y_2}{m_1-m_1}=\infty$$

or the center of gravity of the two filaments lies at infinity. Their velocities = their angular velocity x by the distance from the center of gravity differ from each other by an infinitely small quantity and can be expressed by

$$\frac{\rho_1+\rho_2}{2}\frac{d\theta_1}{dt}$$

but by our preceding equation giving the value of $\frac{d\theta_1}{dt}$ this is

$$=\frac{1}{2\pi}\frac{m_1}{\rho_2-\rho_1}$$

the direction of the motion is, of course, perpendicular to the line giving the cen-

ters of gravity of the two filaments. The particles of fluid lying between the filaments move forwards in the same direction as do the filaments, the one-half way between them moving four times as fast. Let us suppose that the vortex filaments at the beginning of the motion lie on the axis of x at equal distances from the origin, then the particle above referred to will lie at the origin. Also write $\frac{\rho_2-\rho_1}{z}=a$ the absolute distance of each filament from the origin. We have then for the co-ordinates of the filaments at the time t (a, y') and, by virtue of what has been said the co-ordinates of the particle originally at the origin will be $(0,4y')$. The equations of the lines joining these two points are

$$\begin{vmatrix} x, & y, & 1 \\ o, & 4y', & 1 \\ \pm a, & y', & 1 \end{vmatrix} = o$$

The intersections of these lines with $y=o$ are given by

$$x=\pm\frac{4a}{3} \text{ or } x=\pm\tfrac{4}{3}(\rho_2-\rho_1)$$

that is, the lines joining the particle half way between the filaments with the centers of gravity of the same pass through fixed points on the line joining the original positions of the centers of gravity of the filaments. The points lie outside of the original positions of these centers of gravity and at an absolute distance from

$$\text{them}=\tfrac{1}{3}(\rho_2-\rho_1)=\tfrac{2}{3}, \frac{\rho_2-\rho_1}{z}.$$

The particles of fluid that lie in the plane bisects at right angles the line joining the two vortex filaments will remain in this plane. If this plane be considered as a fixed boundary, we have by considering one of the filaments the case of a filament moving parallel to a fixed plane which limits the extent of the fluid.

Let us now assume that the vortex filaments are so arranged as to form the continuous surface of an elliptic cylinder of finite cross section, and further assume ζ as constant for every point of this cross section. As the same particles of fluid constantly remain in any vortex filament, the bounding ellipse of the cross section of the fluid will always be composed of the same fluid particles.

The equation of this line will be a function of x, y and t, and may be written for the moment as

$$f(x,Y,t)=o.$$

Then by virtue of the above we have generally

$$\frac{df}{dt}+\frac{df}{dx}\frac{dx}{dt}+\frac{df}{dy}\frac{dy}{dt}=o.$$

or

$$\frac{df}{dt}+u\frac{df}{dx}+v\frac{df}{dy}=o$$

But,

$$u=\frac{dW}{dy}, v=-\frac{dW}{dx}.$$

Therefore this equation becomes

$$\frac{df}{dt}+\frac{dW}{dy}\frac{df}{dx}-\frac{dW}{dx}\frac{df}{dy}=o.$$

Now the general equation of our ellipse is

$$f=\alpha x^2+2\beta xy+\gamma y^2-1$$

when α, β, γ are functions of t. Assume another system of co-ordinates coinciding with the axes of the ellipse and also passing through its center; then

$$x'=x\cos\theta+y\sin\theta$$
$$y'=-x\sin\theta+y\cos\theta.$$

Call a and b the semi-axes of the ellipse then we have for its equation,

$$\frac{x'^2}{a^2}+\frac{y'^2}{b^2}=1.$$

Substituting for x', y', their values as given in terms of x and y, and this becomes,

$$x^2\frac{b^2\cos^2\vartheta+a^2\sin^2\vartheta}{a^2b^2}+2xy\frac{[(b^2-a^2)\cos\vartheta\sin\vartheta]}{a^2b^2}$$
$$+y^2\frac{b^2\sin^2\vartheta+a^2\cos^2\vartheta}{a^2b^2}=1$$

Comparing this with

$$\alpha x^2+2\beta xy+\gamma y^2=1$$

and we obtain,

$$a^2b^2\alpha=b^2\cos^2\vartheta+a^2\sin^2\vartheta$$
$$a^2b^2\beta=(b^2-a^2)\cos\vartheta\sin\vartheta$$
$$a^2b^2\gamma=b^2\sin^2\vartheta+a^2\cos^2\vartheta$$

In these a and b are constant, but α, β, γ, ϑ are functions of t. Now W satisfies the equation

$$\frac{d^2W}{dx^2}+\frac{d^2W}{dy^2}=-\zeta$$

for all points in the interior of the ellipse, and its value is obtained by integration of the equation

$$\frac{\mathfrak{Z}}{2}ab\int_0^\infty \frac{1-\frac{x^2}{a^2+\lambda}-\frac{y^2}{b^2+\lambda}}{\sqrt{(a^2+\lambda)\ (b^2+\lambda)}}\ .\ d\lambda$$

The integral of this is

$$\frac{\mathfrak{Z}}{2}ab\left(2\log\frac{1}{a+b}+1\right)-\frac{\mathfrak{Z}}{a+b}\ (bx'^2+ay'^2)$$

or, since we only use the derivatives of W, we may write it,

$$W=C-\frac{\mathfrak{Z}}{a+b}\ (bx'^2+ay'^2)$$

for all interior points, and for points at the boundary. If now we write for brevity,

$$A=b\cos^2\Theta+a\sin^2\Theta$$
$$B=(b-a)\cos\Theta\sin\Theta$$
$$\Gamma=b\sin^2\Theta+a\cos^2\Theta$$

we have

$$W=C-\frac{\delta}{a+b}(Ax^2+2Bxy+\Gamma y^2).$$

Let us examine again the condition that we obtained for the bounding line of the cross section of the cylinder, viz:

$$\frac{df}{dt}+\frac{df}{dx}\frac{dW}{dy}-\frac{df}{dy}\frac{dW}{dx}=o$$

we have

$$\frac{df}{dt}=\frac{da}{dt}x^2+2\frac{d\beta}{dt}xy+\frac{d\Theta}{dt}y^2$$

$$\frac{df}{dx}\frac{dW}{dy}=-(\alpha x+\beta)(\Theta Bx+\Gamma y)\frac{4\delta}{a+b}$$

$$-\frac{df}{dy}\frac{dW}{dx}=(\beta x+\Theta y)(Ax+By)\frac{4\delta}{a+b}.$$

Equating to zero the co-efficients of x^2 xy, and y^2 separately we have

$$(a+b)\frac{da}{dt}=4\mathfrak{Z}(\alpha B-\beta A),$$

$$(a+b)\frac{d\beta}{dt}=2\mathfrak{Z}(\alpha\Gamma-\gamma A),$$

$$(a+b)\frac{d\gamma}{dt}=4\mathfrak{Z}(\beta\Gamma-\gamma B).$$

If Θ cannot be determined as such a function of t as to satisfy these equations then will our equation of condition

$$\frac{df}{dt}+\frac{df}{dx}\frac{dW}{dy}-\frac{df}{dy}\frac{dW}{dx}=o$$

hold for all points in the cross section.

Forming the derivatives of α, β, γ with respect to t and transforming them by reference to the values of A, B, Γ we find that the function of t sought is given by the equation

$$\frac{d\Theta}{dt}=2\mathfrak{Z}\frac{ab}{(a+b)^2}$$

We have thus the value of the angular velocity with which the cylinder rotates around its axis. The rotation of the cylinder also induces relative motions among the component vortex filaments. These are obtained by regarding x' and y' as functions of t. We have by differentiation

$$\frac{dx'}{dt}=y'\frac{d\Theta}{dt}\quad\frac{dy'}{dt}=-x'\frac{d\Theta}{dt}$$

and the other components of the velocities in the directions of x' and y' are

$$\frac{dW}{dy'},\quad -\frac{dW}{dx'}$$

Therefore we have by combining these

$$\frac{dx'}{dt}=\frac{dW}{dy'}+y'\frac{d\Theta}{dt}$$

$$\frac{dy'}{dt}=-\frac{dW}{dx'}-x'\frac{d\Theta}{dt}$$

Now,

$$\frac{dW}{dy'}=-\frac{2a\mathfrak{Z}}{a+b}y',\ \text{and},\ \frac{dW}{dx'}=-\frac{2\mathfrak{Z}bx'}{a+b}$$

and

$$\frac{d\Theta}{dt}\mathfrak{Z}=2\frac{ab}{(a+b)^2}.$$

Therefore

$$\frac{dx'}{dt}=-\frac{2\mathfrak{Z}a^2}{(a+b)^2}y',\quad\frac{dy'}{dt}=\frac{2\mathfrak{Z}b^2}{(a+b)^2}x'.$$

Differentiate each of these for t and with

$$\theta=2\mathfrak{Z}\frac{ab}{(a+b)^2}$$

and we have after integration of the resulting well-known powers

$$x'=a\,l\cos(\theta t+i)$$
$$y'=b\,l\cos(\theta t+i)$$

when l and i are the constants of integration and determine the particle of fluid to which x' y' have reference. $\rho>1$ because then for the cases when $\cos(\theta t+i)=1$ we should have $x'>a$ which cannot be, $\therefore$ l is a proper fraction. θ of course denotes the angular velocity of the cylinder, or

$$\frac{d\mathcal{S}}{dt}=\theta$$

from which $d=\theta t$.

Solving the equations

$$x'=x\cos\mathcal{S}+y\sin\mathcal{S}$$
$$y'=-x\sin\mathcal{S}+y\cos\mathcal{S}$$

for x and y gives

$$x=x'\cos\mathcal{S}-y'\sin\mathcal{S}$$
$$y=x'\sin\mathcal{S}+y'\cos\mathcal{S}$$

Substituting for x', y' and $\mathcal{S}$ their values these become

$$x=al\cos(\theta t+i)\cos\theta t - bl\sin(\theta t+i)\sin\theta t$$
$$y=al\cos(\theta t+i)\sin\theta t + bl\sin(\theta t+i)\cos\theta t$$

by expanding the quantities cos sin $(\theta t+i)$ and collecting the terms these equations may be written

$$x=\frac{a+b}{2}l\cos(2\theta t+i)+\frac{a-b}{2}l\cos i$$
$$y=\frac{a+b}{2}l\sin(2\theta t+i)-\frac{a-b}{2}l\sin i$$

by differentiation with respect to t we obtain

$$\frac{dx}{dt}=-(a+b)\theta l\sin(2\theta t+i)$$
$$\frac{dy}{dt}=(a+b)\theta l\cos(2\theta t+i)$$

from which

$$\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}=(a+b)\theta l=2\mathcal{Q}l.\frac{ab}{a+b}$$

and also,

$$\left(x-\frac{a-b}{2}l\cos i\right)^2+\left(y+\frac{a-b}{2}l\sin i\right)^2=\left(\frac{a+b}{2}\right)^2l^2$$

From these equations we see that each vortex filament moves in a circle with uniform angular velocity, the time of rotation being evidently $\frac{\pi}{\theta}$; and that the position of the center and the radius of the circle varies for different filaments. Suppose one of the semi-axes a or b to be infinitely greater than the other; this gives $\theta=o$, and consequently $\mathcal{S}=o$, or the straight line which has become the limiting case of the ellipse does not rotate. If $a=b$ our ellipse becomes a circle, and we have $\theta=\frac{\mathcal{Q}}{2}$; in this case there is no change of the relative positions of the vortex filaments, but they all rotate around the common central axis with angular velocity $\mathcal{Q}$.

THE TREATMENT OF IRON TO PREVENT CORROSION.*

By Professor BARFF, M. E.

From "Journal of the Society of Arts."

It is now more than two years ago since I had the honor of introducing to your notice in this room a process for the prevention of corrosion in iron; and it was through the Society of Arts that it was first made known to the public. My paper then met with a very flattering reception, both from this Society and from the public press. An article appeared in the *Times*, which, written with great ability, gave to it a claim to public consideration, and with one or two trifling exceptions, it was well spoken of in all directions. The exceptions to the general desire manifested to receive well a process which, if practicable, was highly desirable, as being likely to increase largely the use of iron, and to enable it to replace, for certain uses, other metals which were injurious to health, were not founded on reasons of sufficient force to cause me to notice and answer them; and some of them, which could only result from the inner consciousness of those who made them, without an atom of proof, have been fully answered by the work which has been subsequently carried out on a larger scale at my laboratory at Kensington. I was blamed by some for bringing out my invention too soon; but I think that you will see that, by following the course I did, I was enabled to

* A Lecture before the Society of Arts.

give specimens to those who have very thoroughly tested for a long time the goodness of the process; so that I, to-night, stand before you armed with a large amount of experience, and with testimony from others on whose word and judgment reliance can be placed. In answer to some remarks made by the Chairman on the night when I read my first paper, Admiral Selwyn stated that the black oxide of iron had stood the action of sea water for ages, and he therefore advised us to look to nature for a proof of its enduring properties, on the shores of New Zealand, where quantities of it had existed unchanged since the creation of the world. No one, who has a right knowledge of its properties, could doubt its power to resist atmospheric influences, and even the action of sea water; but the doubt that did exist in the minds of many was whether it could be produced artificially on iron, so as to keep its place, and enable the iron beneath it to resist their action as well; or, rather, I should say, whether its adherence to the iron was so complete and perfect as to protect it from them. Pieces of iron will be shown you which were exhibited on that occasion—which have been exposed ever since, and you will be able to judge for yourselves whether the protection afforded by the black oxide is complete and perfect, or not. I feel that I ought, before proceeding further, to notice briefly one or two of the exceptions to which I have alluded in the reception which my first paper met with. One was that the process was not original; that I had no right to claim as my discovery what had been known to chemists for years. I will quote from my paper, and then you will see whether I deserve this charge or not. "In every school where chemistry is taught, in the most elementary lecture on hydrogen, the pupils are told that, if they pass steam over red-hot iron filings, contained in an iron tube, they will be able to collect and burn the hydrogen gas at the opposite end of the tube to where the steam enters. For a long time it was thought that the particles of black oxide formed by this decomposition of the steam were pulverulent, and could not be made to cohere into a solid mass." It is manifest that I could not claim as my invention what I stated was already known. Another exception was that the process might be of use for small articles—pots, pans, &c.,—but that it could not be applied to large articles, and, even if it could, it would so materially weaken the iron that dependence could not be placed on its strength; in fact, if I remember rightly, a solemn warning was given to persons not to trust to it. Now, that the process is only applicable to pots and pans, &c., the articles before you will disprove. At the beginning of my experiments—I did not wish to incur a great outlay, and, therefore, the chambers, or muffles used were not large. A year and a half ago, I had a chamber built of fire brick, and that has been in use ever since. In it articles six feet long have been treated; and if the chamber were 12 feet long, or 20, articles of such lengths could be treated as well as those which you see before you. As to the action on the strength of the iron, bars treated have been tested for breaking and tensile strain, and the result is that the strength of the iron is not affected, and the persons who tested them assert that they would not hesitate to use the process because of any injurious effect which it has on the strength of iron. I need not do more this evening than briefly remind you that my process consists in oxidizing the surface of iron by means of superheated steam. In my former paper a description of the rusting of iron will be found; it is enough for my present purpose to state that the black oxide of iron is unaltered by any of the ordinary influences which produce red rust, and which therefore cause the destruction of iron.

The points which I have to bring before your notice this evening are those which two years' experience has enabled me to discover in the method of working the process; and these are very important, because formerly there was a want of certainty in performing it which gave very unequal results. During the last eighteen months I have been able to give constant attention to it, which I was not able, from my engagements, to do before, and now I can assert that it can be conducted with ease and with perfect certainty. In the specification of my patent, the method of performing the necessary operations is given, but considerable practical experience is required, which it

is impossible to describe in writing. In the earlier experiments performed at my laboratory at Kilburn, it was often found that the coating of black oxide scaled off wrought-iron articles. This is never the case now.

This scaling resulted from an insufficient and irregular supply of steam to the muffle during the operation, whereby air was not excluded, but was often forced in from the want of a sufficient pressure of water on the superheating pipes. Air must be completely excluded from the oxidizing chamber, because, if the oxidation of the iron depend, during any part of the process, on the oxygen in the air, such oxide formed will not adhere to the iron properly. This I have proved by submitting iron to oxidation by dry air, and in every case wrought-iron has, when so treated, lost its coating, which has flaked off in scales; and in the case of cast-iron, the oxide on exposure comes off in a very short time, and therefore does not provide perfect protection to the iron. If, however, the air forced into the chamber be moist, the same result occurs with wrought-iron, but with cast-iron the coating formed does adhere for a time, and the length of its adherence is proportionate to the quantity of moisture present in the air. If the air be forced into the ordinary chamber from a vessel in which it is in contact with water, and if the temperature of the room in which this vessel is be high, as in such a case it must be, the quantity of moisture converted into steam, when at the temperature of the iron to be oxidized, will be great, in fact, enough to oxidize the iron, for very little steam is required to oxidize a great weight of iron; but then the oxygen of the air will take part in the action, and wherever the iron is oxidized by the oxygen of the air its adherence will not be complete, and though by being mingled with the other oxide it may have a certain amount of stability, yet in a short time it will come off. I exhibit two specimens in illustration of this, one of cast the other of wrought iron, both of which have been exposed in the open for some time; the piece of cast iron did not rust for some time after it was exposed, but the wrought iron flaked and rusted at once. It appears, therefore, to be absolutely necessary to secure a good result, that air must be completely excluded from the oxidizing chamber.

For a long time I experienced considerable trouble from the appearance of small spots of rust on articles otherwise well coated, which were immersed in water. The spots of rust appeared to increase in size, but on examination it was found after washing off the rust, which could be easily removed, that it originated from small openings in the coating of black oxide. It required a magnifying glass to see these openings; the rust did not spread by more of the iron surface rusting, but because the rust formed in these minute cracks was carried out by the water in which the articles were, and was therefore diffused about. Such rusting has no effect on the strength of the iron, and after a few cleanings it ceases altogether. However, I felt that it was very necessary to prevent it, and that led me to seek carefully for its cause. When iron is heated it expands, when cooled it contracts. If iron be heated in an oxidizing chamber it expands, its pores, so to speak, open. If a jet of superheated steam be admitted at a temperature lower than that of the iron in the chamber, the iron will contract, and then will decompose the steam—of course it must be at a sufficiently high temperature to do so. Now, the iron will gradually get hotter, and will expand again, and the first thin coating of black oxide will be in part cracked, and as the oxide goes on forming it will in part cover and fill up these cracks, but I think—in fact I am sure—that it does not do so perfectly, and hence some of them remain, the iron at the bottom of them being coated with but a very thin film of oxide. Reasoning in this way, I came to the conclusion that no contraction must be allowed to take place in the iron after the oxidizing action had commenced, and to secure this, the ordinary chamber is always kept at a much lower temperature than the superheater; and now it is never allowed to rise above five or six hundred deg. Fahr. before the superheated steam is admitted, and the steam is never allowed to pass in at a temperature less than one thousand degrees. This for a long time has been our invariable plan of work, and in no case whatever have we experienced any failure as long as the apparatus was sound. Many have tried

experiments, independently of me, but in all cases I have heard complaints that they have not succeeded, and I feel sure that the want of success has been due to one of the causes I have mentioned, or to another which I described in my former paper, viz., the presence of moist steam.

When asked by my lamented friend, your late able Secretary, to read another paper on my iron process, I willingly consented to do so, because it would give me the best method of making public and explaining the difficulties I had met with and overcome, but which were still troubling others who take a real interest in the process. It has always been my opinion that the best way of forming the black oxide on iron is to conduct the process by means of superheated steam alone, because the steam, being the source of heat to the iron, raises its temperature to that at which it can decompose steam, so that oxidation commences immediately the iron is hot enough. When the iron is heated in the chamber, before the steam is allowed to act upon it, there is always danger of air getting into the chamber and forming a film of oxide before the steam gets to work, and this is a thing to be avoided. I have only been able to experiment with superheated steam alone on a small scale, and the large chamber has flues up its sides which would conduct off the heat if it were attempted to raise its temperature by superheated steam alone. I may be here misunderstood. The flues at the sides of the chamber would cause cold air to circulate round it, and the heat from the superheated steam would thus be conveyed away. The experiment I did perform was with an iron muffle, similar to that which was used in the early experiments. This was surrounded with fire-clay, to act as a non-conductor of heat. Steam at 1,500 deg. Fahr., was injected into it for a short time, and then the articles to be treated were put inside it, and the steam was again let in. In a short time the muffle and its contents became red hot, and, after a few hours, were found to be well coated with black oxide. I could not work this process on a larger scale, for I have already, through the assistance of a friend, expended a large sum of money in experiments, which have resulted in my being able to state that my process is now commercially perfect, and is waiting the enterprise of gentlemen in the iron trade to take it up and use it. When I last had the pleasure of addressing you I spoke with diffidence, and I could not give definite answers to those who questioned me; now I can speak with confidence, and though I cannot perhaps reply to all questions to your entire satisfaction, yet I can assert that there is no reason why this process should not be largely adopted. Adopted it will be, for it is a success, and has been proved to be so by the testimony which I have the pleasure of submitting to you. But, before submitting these testimonials, allow me to say a few words about the properties which this artificially-formed black oxide possesses, as to which I could not testify in my first paper. It gives great hardness to the surface of iron, when the coating is sufficiently thick; by this I mean when it is even less than one-sixteenth of an inch. An ordinary flat rasp will not remove it without great labor; it resists emery powder, as I stated in my first paper; but now I have proved that it will for a long time resist a rasp, and will remove pieces of steel from it. This has been witnessed by many, among these by M. St. Yves, Engineer in Chief of the Ponts et Chausées, Paris, who was sent over to report on the process, and Professor Frankland. Substances which adhere to iron, zinc and enamel will not adhere to it. Saucepans in which arrowroot and other sticky substances are cooked can be cleaned with the greatest ease, after they have been oxidized, a simple wipe removing all dirt. I exhibit a saucepan which has been in use at my house for two years, and a wrought iron stew-pan which has done about six months' service. I have a whole set of stew-pans at home, and my cook prefers them much to any others. Dr. Mills, of Glasgow, testifies to the same property. You see before you a urinal which was in constant use at my laboratory for months, and was then sent to be used here two months ago. There are no deposits on it. I had water evaporated in an oxidized pan for six weeks—common tap water; the water never boiled, but was slowly evaporated. The deposit found was removed with a duster; it did not stick to the iron. This

is a matter of great importance to boilers, and for pipes through which water is to be conveyed.

Now, articles coated can be submitted to a high temperature, even a red heat, without the coating being injured or disturbed.

I have written to most of the gentlemen to whom specimens were sent two years ago, after my first paper, and most of them have kindly replied. Their letters I have published, even those which show that at present my process does not meet their requirements. Where it has not answered, I have added in a note my own remarks for what they are worth. At present I fear that iron wire cannot be treated successfully—the wire can be treated and will not rust, but it cannot be bent to a sharp curve without the coating coming off. I show a specimen to prove that the wire, when not bent, does not rust, and that articles made of wire can be made non-rustable provided they be not stretched beyond a certain point. Riveted iron plates can be most successfully treated; the process tightens the rivets and assists the caulking; the plates before you show this. I have not solved the question of riveting plates after treatment, but I am sanguine that I shall be able to do so. Weights are treated for the Government, and submitted by Mr. Chaney to tests, and the process is now recommended by that Department for the standard weights throughout the country. I also exhibit two specimens, one of oxidized, and the other of common iron, on which gold leaf has been put in the ordinary way, with oil gold size, and I think they illustrate well that even where it is desired to paint or gild iron to be placed in exposed situations, it is very desirable to have it first treated by my process; both the specimens have been out of doors for two months, exposed to rain and snow; for some days they were completely buried in snow.

I regret to say, gentlemen, that I cannot speak very definitely as to the cost of the process. I do not wish to delude any one by a statement that it can be done for so much per ton. It is simply impossible to do this, as you will see. Hollow goods—such as saucepans, &c.—will take up a much larger space per ton than a ton of 56 lbs. weight, and this shows how fallacious any general statement on this head must be. My experiments have not been conducted with special regard to economy, but to efficiency, and, having settled this point, economy must now be inquired into. This is rather the work of the manufacturer than mine; but this much I can say for your guidance, that, even with my means, the cost for light articles does not exceed that of galvanising. But I do think that, if the treatment gives a permanent protection to the iron, that articles treated by it should command a higher price than those which have been treated by a less enduring process. My experience tells me that different kinds of cast iron behave differently under treatment. Some kinds require longer exposure to the action of the superheated steam than others. Why this is I cannot as yet find out. Of this class is the iron in which the carbon is in more perfect chemical combination as a carbide. This iron is whiter than the other kinds. There are before you specimens which have resisted the rusting action of air in the presence of water. The two statues exhibited and other articles are of a different kind of iron; they required a shorter exposure, and have stood equally well. I have not yet met with any sample of cast iron which could not be properly treated. Wrought iron reqires a somewhat different treatment; a lower temperature, about 900 deg. Fahr., suits it best, and steel also. It is not well to expose articles very different in bulk at the same time; all that are put into the muffle should be pretty nearly equal in bulk. I mean that very heavy articles, such as 56 lbs. weight, should not be treated with these gutter spouts. Cast and wrought iron should not be treated together; but all these are matters which a little experience will regulate perfectly. Sometimes the sand from the mould adheres to cast iron; this is often the case inside pipes; it is of no moment, for the sand itself gets so firmly fixed on the coating of black oxide, that it assists in protecting the iron. I have proved this by severe experiments. In clearing off the rust from iron before it is submitted to the action of superheated steam, the usual method is employed, it is immersed in dilute oil of vitriol, and after washing is put into

some bran water; this last operation is to remove any basic sulphate of iron from the surface. If this basic sulphate is not completely taken away when the iron is heated, it is reduced, and red oxide of iron is left on the surface, which has the color of the red oxide used for paints, and you will see some articles so colored before you; this red oxide does not prevent the formation of the black oxide beneath it, and does not interfere with its stability. it is therefore of no importance, except to the appearance of the articles.

REPORTS OF ENGINEERING SOCIETIES.

THE AMERICAN SOCIETY OF CIVIL ENGINEERS.—At a regular meeting of the Society held on the 2d of April, Major-Gen. J. G. Barnard read a paper by the late Gen. B. S. S. Alexander, descriptive of the construction of the *Minots Ledge Light-House*, off Cohasset, ten miles south of entrance to Boston harbor. After mentioning several of the most noted and dangerous rocks throughout the world, upon which light-houses have been built, the paper relates in a most interesting way the first visit of the author to the Minots Ledge, the delays encountered before a reconnoisance upon the rock could be effected, the plan of operations devised while waiting; then the date of commencement, on April 1, 1853; what was done in preparing the foundation, cutting the stones, and laying and securing them; the number of hours worked per annum, ranging from 130 to 377 in the seven successive years that were spent in the construction of the light-house; the rules adopted for doing the most work in the time, and at the least risk of drowning, and the cost $300,000. It was completed June 29, 1860, and the first light was shown on Nov. 15 of the same year. The paper was exceedingly interesting throughout, and was fully illustrated by diagrams, showing the excavation of the foundation, and the plan of laying the stones and securing them against failure.

Gen. Barnard made some remarks upon the settlement of Pier No. 2 of the South Street bridge, Philadelphia, a paper upon which, by D. Mc.N. Stauffer, C.E., Philadelphia, was read before the Society, Sept. 4, 1878, and published in the transactions of the same month. General Barnard illustrated his remarks by reference to his own experience with the foundations of Fort Livingston at the mouth of the Barataria River (or bayou) La., and the settlement of the Fort, which, when examined a few years ago by a commission of officers, showed a sinking of nearly four feet. Fort Bienvenu, built in one of the worst marshes in the vicinity of New Orleans, has sunk three or feet feet. Fort Jackson, on the Mississippi River, below New Orleans, had settled perhaps one foot by the year 1840. It was a very heavy work; the parapet and coping were not laid, and, being abandoned for many years previous to the late war in the South, bushes and tall weeds grew up and hid it, which the raising of the levee further aided, so that river pilots used to say that the old fort was sinking out of sight. The sinking of the pier of the South street bridge, General Barnard considered, was occasioned by the escape laterally of the soft mud in the pocket which existed under one corner of the foundation; that the settlement commenced from the date of loading the foundation, and that the lesson to be drawn was to make more numerous borings, and prevent lateral escape of the mud before laying a heavy and costly foundation.

Mr. Dresser remarked: "In the case of the construction of a large hospital in New York, it was supposed that there was solid rock foundation to build on. Excavations developed the fact that under a portion of the building a pocket filled with mud or clay existed, into which the rock sloped suddenly. The question was to so join the foundations upon the solid and the compressible bottom that the settlement in the latter case would not crack the superstructure where compression ceased and solid rock commenced. This was done by excavating a trench in the rock and filling it to the depth of four or five feet with the material taken from the 'pocket.' The structure was completed upon this, and no crack shows itself up to the present time."

Mr. Collingwood gave an account of the removal of pockets of material under the caissons of the East River Bridge, and also of the foundations of piers etc., in the approaches in the "Swamp," as follows: In preparing the bed for the New York caisson, much the same thing was done as was mentioned by the last speaker (Mr Dresser). The bed rock was quite uneven, and it was struck at several points at about two feet above the level finally fixed upon as the stopping place of the edge of the caisson. Over all other parts the sand and gravel varied in depth from one to sixteen feet, as found by borings. The high points in the rock were cut away, so as to leave a space of at least a foot everywhere between the caisson and the rock, and loose sand was thrown into the space. At other points there was found in pockets some fine sand, which we were fearful might move. The sand was taken out from a trench along the edge of the caisson, and this was filled with concrete, so as to cut it off and prevent possibility of movement. No great settlement was expected, but these precautions were taken so as to insure uniformity in settlement, and, as far as we could judge, they were entirely successful.

In founding the New York approach across the "swamp" (so called), considerable difficulty was anticipated, and we were assured by persons acquainted with the locality that piling would be necessary. Careful preliminary borings showed that a sound, clean, sandy bottom could be everywhere reached, with a maximum depth of eleven feet below the level of mean high tide. I decided, therefore, to make an open cut, and remove all compressible material.

Fortunately, there was a thin bed of clay over the sand, and by removing this over a

portion of the bed only at a time, and then concreting it, we had no great difficulty in keeping the water down.

In the last pit the water broke in with considerable force at one spot, but by putting in an interior line of sheeting, and leaving an open way for the water to the pumps, we were able to build up the other parts of the masonry for a few feet, and then by pumping rapidly and inclosing cement in bagging, to get the foundation in over the small uncovered portion. No settlement has since occurred.

PROCEEDINGS OF THE ENGINEERS' CLUB OF PHILADEDPHIA.—*March 15th*, 1879.—Mr. C. E. Buzby exhibited a model of Travers' Iron Railway Tie. It is in use on the Philadelphia & Baltimore Central Railroad, near Lamokin. The device dispenses with all spikes, bolts, nuts or fish plates, and drilling or punching the rails, avoiding fractures from such causes.

The iron tie, it is claimed, will outlast twelve renewals of the ordinary tie at one-half the cost to keep in repair.

Each tie is recessed under its rails, and along the bottom of the recesses wedge-shaped pieces are cast transversely. At the sides of each recess are creosoted blocks, which form a cushion, and a fulcrum for two clamps, which grasp the flange and web of the rail above, bearing upon opposite faces of the wedge below. The weight of the train forces the clamps upon the wedge, spreads them at the bottom, and grips the rail. The first cost is somewhat greater than the wooden tie, but is said to offset this in durability.

Percival Roberts, Junior, announced the death of the Vice-President of the club, Mr. J. B. Knight, and offered resolutions of respect, which were adopted.

O. B. Colton submitted a plan of the contemplated Water Works at the Middle Penitentiary at Huntingdon. Mr. W. A. Cooper read an interesting article on Richardson's method of sharpening and re-cutting files by the sand blast process as used by Trump Brothers, of Wilmington, Delaware. This process, it is claimed, will renew completely worn-out files and rasps at a nominal cost, without the necessity of softening and re-hardening, thereby retaining the original temper. Samples of the work done by this process were exhibited.

At a recent meeting of the Engineers' Club of Philadelphia, at their rooms, No. 10 North Merrick Street, Mr. Charles A. Young read a paper on the Intra-Conglomorate Coals shown on the New River, West Virginia, describing the general geological and topographical features of the district, and presenting sections, analysis of the coal, and a table showing its calorific value were included in the paper. In tee discussion of this paper, Mr. Hardin gave an account of the operations at Sewell Station, Chesapeake and Ohio Railroad, by Messrs. Firmstone and Pardee, descriptive of the means used to carry the coal from the mouth of the mine to railroad grade. Mr. Charles A. Ashburner, through the kindness of Messrs. P. & T. Collins, contractors of the Madeira and Marmore Railway, Brazil, presented to the club an abstract of the report of progress made by C. O. D'Invilliers, Chief Engineer, together with tracings of the maps and profiles of the preliminary and located lines:

"SAN ANTONIO, October 10th, 1878.

"MESSRS. T. AND P. COLLINS:

"*Gentlemen*:—According to your request, that I should, as soon as possible, submit a record of our work and a report of the country we have surveyed, I have had plans made of all our surveys up to date, which comprise the country between San Antonio and Calderao do Infierno, which I respectfully submit in connection with this report.

"Our examinations during the first two or three months, confined principally to the territory between here and Macacos, a distance of five miles by river, developed a country much broken by deep ravines, at the heads of which we found a plateau or ridge dividing the waters of the Jamari from those of the Madeira, which, at different elevations, continues to exist as far as the Jaci Parana River. The ground along the river's edge is flat and good, and, with a low embankment, would be available for railroad purposes, except from the fact that abrupt hills come flush to the river's edge at Macacos, Theotonia Falls and other points.

"Between the river and the dividing ridge there is no alternative but to climb to the plateau at a maximum elevation of 200 feet above high water.

"From San Antonio to the end of the sixth mile there is generally rough and comparatively heavy work; from there to the end of the forty-third mile, or six miles this side of the Jaci Parana River, the country is flat and gently rolling, at an average elevation of 400 feet above high water, over which a very cheap and generally favorable line can be obtained.

"Between the ridge and the Madeira River are numerous detached hills, many of them quite high, into which we ran with some of our trial lines. These hills are generally in the neighborhood of the rapids. Calling high water 200, the maximum elevation of the ridge is 400 feet, that of the detached hills 500 feet.

"Our survey along the Jaci Parana River shows us a very crooked stream from 350 to 400 feet wide during low water, with generally low banks from 2 to 8 feet below high water, and in some places high bluffs on one side and lower ground on the other. The highest and best ground on the west side is that which we have chosen for the crossing; it is about 8 miles from the mouth of the river by the river, and four miles in a direct line.

"From the crossing toward Santonio the ground rises gradually about 2 to 3 feet per hundred until it reaches the plateau, which is 100 feet above high water, being level in a direction up and down the Jaci Parana.

"Regarding water ways between San Antonio and Calderao, we shall require nothing in the bridge line except over a creek one-half mile from San Antonio and the Jaci Parana River. The creek is 150 feet wide at high water. Over the river from 350 to 400 feet of bridging will be ample. The necessary cul-

verts will average one in every two miles. As for timber, we can get tie timber anywhere. The timber is the heaviest on the first twenty miles, and gets gradually lighter as we approach the Jaci Parana River. Adjacent to the river small palms mostly abound, few of the trees being more than 10 inches through. The brush is everywhere very dense, making it impossible to investigate the country on either side of the line without running cut lines and levels.

"We have been much thrown back on ac count of sickness among the corps, resulting not so much from the unhealthiness of the cli mate as from the difficulty of keeping up the supplies of proper food. My own experience has been, that beyond a slight bilious tendency, the climate is not unhealthy, except, perhaps, through the months of July and August.

"There has been no suffering from the heat by those in the woods, although the temperature ranges at about 100° Fah. (max.) in the shade, and 120° (max.) in the sun. As to the geological features, I am inclined to think we shall find no solid rock, except at stations 110 and 200. In nearly every cut in the first six miles we have found a decomposed granite cemented with clay or iron—in many cases, granite boulders. That composing the Cachuelas is undoubtedly of volcanic origin.

"Regarding the location of our line to the Madeira River, after passing Macacos, where we are 2006 feet in the closest point, is at the sixteenth mile post, or two miles above Rosstown, where we are about two miles in. At Theotonio Falls, ten miles from here, we are six miles in; at San Carlos, twenty miles from here, three miles in; at the thirty first mile post, we are most distant, being nine miles inland; at Jaci Parana River, four miles; at Calderao, two and one-half miles; at Tres Irmaos we shall probably be on the river bank.

"I am, gentlemen, your obedient servant,

"C. S. D'INVILLIERS, *C. E.*"

"NOVEMBER 25, 1878.

"MESSRS. P. AND T. COLLINS:

"*Gentlemen:*—I send you this month maps of the located line of railway from San Antonio to station 1100, or about twenty-one and one-half miles, and projected location from station 1100 to Calderao do Infierno or station 3530. The locating party are now about station 1700, and making very rapid progress.

"At the time of writing, the track is laid to station 208, or four miles, and the grading from there to the end of the six miles of heavy work more than half finished. Considerable of the track is laid on temporary line at an expense of about $8,000.

"There have been nine trestles erected (two on temporary line) of a total length of 1453 feet, which have cost $7,500. There are twenty-one miles cleared of timber.

"In order to obtain the information we have been obliged to run about 160 miles of trial line.

"The locating party under Mr. Byers is composed principally of Bolivian Indians, there being seven whites and twenty-five Indians. These do the duty of axemen, move the camp, and transport provisions from San Antonio.

"On the first six miles there will be 158,500 cubic yards of excavation, about two-thirds of which will be paid for as rock or shale, and 175,000 cubic yards of embankment. On the remaining distance the quantities will probably average from 10,000 to 15,000 cubic yards of material to be moved per mile. There will probably be required, on the average, one culvert per mile, for which I should propose using iron pipe.

"The only bridging we are sure of is 150 lin. feet at station 30, and 350 lin. feet at station 2740, over the Jaci Parana River, though we may have to bridge some of the smaller streams—at most, three or four of them. There is but little stone to be obtained anywhere, except at San Antonio and Theotonio Falls. At the latter place the stone is already cut and bedded by nature.—Your obedient servant,

"C. S. D'INVILLIERS, *Chief Engineer.*"

"Mr. George Burnham, Jr., read a paper on the subject of some Features of Ancient Engineering." Modern research has developed the fact that nearly all the materials (in a very wide sense of the word) of modern civilization originated in antiquity; the peculiar province of our time being to ring the changes of variety upon these elements and give them an immense diffusion.

The textile fabrics of wool, cotton, flax and silk were known to the Egyptians of three or four thousand years ago, but the cotton-gin, the power-loom and the steam-engine have greatly increased their variety and put them into the hands of every one. The same thing is true of the Engineering art for, if we except iron framing the ancients originated nearly all the typical forms we now employ. They were acquainted with the constructive uses of wood; carried stone construction to a point that we have never since reached and probably never shall; their *brick* work dates from the very earliest times, and they constructed canals and aqueducts for irrigation, water supply and inland navigation as well as elaborate drainage systems long before their civilization culminated.

The Chaldean structures, dating from 2200 to 1500 B.C., were built of small sun dried bricks laid in bitumen and faced with kiln dried bricks stamped with the name of the king. These temples were built on elevated platforms of beaten clay in some instances cased with massive walls of stone the object being to raise them above the level of the plain for architectural effect and to avoid inundation. A brick burial vault at Mugheir exhibits a rudimentary arch. The vault is seven feet long, five feet high, and three feet seven inches wide. The sides slope gently outwards until the springing line is reached when the successive courses are pushed towards each other until they meet at the top.

Similar arches are found in early Greek work at Phigalia, Messene, and other places.

The old notion that the round arch was of Roman and the pointed arch of Gothic origin has been dissipated by the spade of the archæologist. Both of these varieties are found in Assyrian work. They are usually of brick and occur in underground construction as

drains and vaults. The brick arch existed in Egypt as early as 1540 B.C. and a stone arch has been found dating from 600 B.C.

The masonry of the past is, of course, identical with ours since we have simply adopted the methods of the ancients. We find in Egypt and Western Asia smooth and rock faced ashlar, rubble, and irregular range work essentially like that of to-day. The Assyrian and Egyptian bas-reliefs indicate their method of moving heavy masses. Sledges were used drawn by large bodies of men. Rollers were placed under the sledge and the piece was carefully "guyed" by parties of men with appropriate ropes and props.

The Roman military roads crossed mountains and valleys without regard to the nature of the ground; tunnels, open cuts, embankments, and bridges frequently occurring. Place cross ties and steel rails upon a Roman road and suppose the grade not too steep, and the points of approach and divergence of modern and ancient engineering are at once apparent. Substantially the substructure was the same as that of a modern rail road, but in place of the pedestrian or the ox team we have the locomotive with its "Fast Express" or heavily laden freight train.

Prof. L. M. Haupt read a paper explaining the *modus operandi* of the "Holly System of Steam Heating" following it by some remarks concerning its economy, general application and rapid introduction.

In his introduction he laid great stress upon the necessity for providing more perfect ventilation and the difficulties presented by the prevailing system of hot air furnaces.

The plant of the Holly Company is very similar to that generally used for the distribution of other fluids, as gas or water, that is there are generators or reservoirs where the steam is made under a pressure of about sixty pounds. Thence it is conveyed by mains to the house or shops where it is to be used at a very low pressure. It is registered by a meter in each house and can be turned on or off at pleasure. New radiators have been introduced open at the bottom so that the air is kept sufficiently moist and the racket occasioned by the condensation entirely avoided.

It eliminates the necessity of keeping up so many different fires, avoids all dust and removal of ashes, and is far more economical than the present system. It has been used with great success to warm schools, churches, halls and dwellings, to run mills and machinery, to extinguish fires, for cooking, heating water and for many other purposes.

The ingenuity displayed by Mr. Holly in so successfully solving the many mechanical difficulties connected with the transmission of steam for general use places him in the foremost rank of inventors and makes him a public benefactor.

The system has been in practical operation for two years in Lockport and organizations are now rapidly making for its introduction in many other places. It is in use in Buffalo, Detroit, Utica, Rochester and other cities and is just being introduced into New York City, Troy and elsewhere.

April 5th, 1879.—Prof. L. M. Haupt read a paper on the "Nomenclature and Classification of Masonry."

Various discrepancies exist in the use and meanings of the terms employed to designate the classes of work, and to remedy this Mr. Haupt attempts to remove the ambiguities by comparing all the authorities and arranging them in such order that they can be more readily referred to and understood.

Mr. Cleeman spoke on the subject of "the proper amount of water-way for Culverts," presenting the formula of Major E. T. D. Meyers of Richmond, Va., which in practice on some railroads recently built in that state appears to be more applicable than the formula heretofore used.

Prof. Marks, University of Pennsylvania, exhibited his adaptation of Peaucellier's Compound Compass, to the drawing of circles of any radius, and mathematically straight lines. This compass was built by Messrs. W. J. Young & Sons, Philadelphia, for the State Geological Survey.

IRON AND STEEL NOTES.

PROPOSED GENERAL CLASSIFICATION OF IRON AND STEEL.—This proposition emanates from a sub-committee of the German Railway Union, appointed for the purpose, and takes the form of a report presented to the general meeting of the Union at the Hague, July 1877. The meeting agreed to act on this report, both by bringing influence to bear on various governments, with the view of having such a classification definitely fixed and agreed upon, and also by instituting further experiments in order to render the classification more exact.

The sub-committee's report first enlarges on the desirability of such a classification, and especially mentions the great inequalities now found to exist among different specimens of iron and steel, supposed to be of similar make, owing to an ignorance of the laws under which differences of quality are produced. It observes that such a classification might require revision from time to time, and then proceeds to give a table which it considers to be as perfect as can be attained in the present state of knowledge. The two qualities specified throughout are strength, as represented by the tensional breaking strain in kilogrammes per square centimeter, and ductility, as represented by the percentage of contraction of area at the breaking point.

The table is as follows:

(*See Table on following page.*)

The above classification would not of course do anything towards solving the question which class of material is the most fitting to use for any particular purpose. This requires a much more extensive range of experience and of tests, and the report concludes by strongly recommending that an Office of Research (Versuchsanstalt) should be established by government with the special object of investigating such matters.

	Tensional Strength. Kilogramme per Square Centimeter. = .00635 Ton per Square inch.	Ductility. Contraction per cent.
A. Bessemer steel, cast steel, or Siemens-Martin steels used as a material of construction for rails, axles, tires, &c.		
1st Quality { Class *a.* Hard....	6,500	25
" *b.* Medium.	5,500	35
" *c.* Soft....	4,500	45
Fracture to be uniform, and no cracks visible in the broken ends.		
2nd Quality { Class *a.* Hard..	5,500	20
" *b.* Soft...	4,500	30
Fracture, &c., as above.		
B. Bar iron.		
1st Quality.....	3,800	40
2nd "	3,500	25
C. Plate iron.		
1st Quality { with the grain...	3,600	25
across " ..	3,200	15
2nd Quality { with the grain...	3,300	15
across " ..	3,000	9

All material which falls below these limits it is proposed to distinguish as "non-classified" material.

RAILWAY NOTES.

THE State of Guatemala, the largest in extent of territory and population in Central America, has recently granted a concession for the construction of a railroad from the port of St. Jose, on the Pacific Coast, to the city of Escuintla, in the interior. Escuintla is a town of some 8,000 inhabitants, situate about thirty miles from the port of St. Jose on the high road to the capital, from which it is nearly an equal distance. Guatemala, the capital of the State, contains a population of 50,000 to 60,000, is a well-built, old established city, and is the residence of the foreign ministers to the Central American States. It is intended to continue the construction of this line to the capital city of Guatemala, and eventually to the Atlantic port of Santa Tomas, and, if thus extended, this line must become a new inter-oceanic route. In this relation the Guatemala line will, says the *San Francisco Bulletin*, have the advantage of a shorter sea route between San Francisco and the Atlantic cities, of from 1,500 to 2,000 miles, and also a large local trade in which the Panama Railroad line is altogether deficient.

AN electro-magnetic railway ticket counter, to indicate the number of passengers going by a particular train, and so the number of carriages of each class required, has recently been adopted at the station of the Kaiser Ferdinand's Nordbahn, in Vienna. The date press of each of the three ticket boxes is furnished with an electric contact, actuated each time a ticket is pushed in to be stamped. Three lines connect the presses with a clock-shaped counter in the office, where the train is arranged, having discs with numbers and pointers, so that the number of passengers booked can be seen at once. The arrangement conduces to promptness in ordinary circumstances, and where great crowds are travelling successive trains can be quickly despatched without overloading with passengers. Messrs. Meyer & Wolf of Vienna, supplied the apparatus.

WE take from a detailed review of the railways constructed in the United States in 1878, published in the *Railroad Gazette*, the following condensed statement of the mileage of new railroad constructed in each state and territory during the year 1878:—Alabama, 22; Arizona, 30; Arkansas, 7; California, 71½; Colorado, 193¼; Dakota, 15; Delaware, 6; Georgia, 62; Idaho, 126; Illinois, 103, Indiana, 74; Iowa, 255½; Kansas, 169½; Kentucky, 20; Maryland, 5½; Massachusetts, 6; Michigan, 110½; Minnesota, 338½; Mississippi, 26; Missouri, 209; Nebraska, 55; New Hampshire, 35; New Jersey, 3; New York, 129½; North Carolina, 16; Ohio, 97; Oregon, 36; Pennsylvania, 188½; South Carolina, 16½; Tennesse, 10; Texas, 118½; Virginia. 16½; Washington T. 15; West Virginia, 16½; Wisconsin, 83¾; or a total of 2688 miles compared with 7340 in 1872, 3883 in 1873, 2025 in 1874, 1561 in 1875, 2450 in 1876, and 2281 in 1877.

THE fall of a railway bridge took place recently on the Manchester, Sheffield, and Lincolnshire Railway. The bridge, situated at Chalk Hill, was considered to be in great danger of falling, and it was decided to take it down. Last Sunday night was appointed for the work, as no week-day could be spared in consequence of the great traffic at that place. Mr. Fisher, the resident engineer, was on the spot, and at nine o'clock operations commenced. The recent frost had done considerable damage to the earthwork, and the brickwork of the bridge had been also, to a large extent, dislodged. It was decided, if possible, to blow up the bridge, and no less than seventeen shots were placed thereon without making any perceptible effect. It was then determined to take it down piecemeal. The workmen worked on a temporary stage a few feet above the bridge proper, and were perfectly safe. About half past three o'clock on Monday morning, a little less than half of the key of the arch had been removed, when without a moment's warning the bridge fell. Some of the men had unfortunately, contrary to orders, been standing on the key of the arch a little before, and, as a matter of course, they fell with the bridge and were buried in the *debris*. Great efforts were made to get them out alive, but four of them were quite

dead when found. Twelve other men were more or less injured, and three were taken to the Grimsby Hospital, and the remainder to their homes.

EUROPEAN MOUNTAIN RAILWAYS OPERATED BY LOCOMOTIVES WITH COG-WHEEL DRIVERS.—The Journal of the Hanover Society, for last year, contains the report of a lecture by one of its members, Mr. Grove, on the several mountain railways built in Austria and Switzerland within the past eight years. The list (adding the Mount Washington Railway for comparison) is as follows :

Date.	Name.	Gauge.	Length. Kilometer.	Maximum grade. Rise in 1000	Least radius of Curvature. Meter.
1870	Ostermundigen, near Bern	Common.	0 56-100	100	No curves.
1871	Vitznau-Rigi, Switzerland	"	7	250	180
1874	Kahlenberg, near Vienna	"	5 16-100	100	180
1874	Schwabenberg, near Pesth	"	3	102 5-10	180
1875	Arth-Rigi	"	9 8-10	212 6-10	180
1875	Rorschach-Heisden	"	5 5-10	90	240
1876	Wasseralfingen	1 meter.	8 25-100	78 5-10	400
1877	Rutl	Common.	—	—	—
1869	Mount Washington	"	5 3-10	375	151 5-10

The rack is made of two channel irons, spaced 5 inches, in which plays the cog-wheel of the engine, 4-inch face, and 4-inch pitch of cogs. The Wasseralfingen road alone has a $3\frac{4}{10}$-inch pitch. The cogs are shaped as evolutes of a circle, so that a variation in the depth to which they take into the rack does not affect their regular action. On the Rorschach-Heisden road the rack is elevated above the ties on two stringers, leaving the space open under the rounds of the ladder-like central rail, so that snow may fall through or be pushed through by the cog-wheel. This seems to have worked well, even solid ice having been thus pushed out from between the cogs. The latest roads have the rack fastened to the ties by suitable castings instead of by longitudinal stringers. Switches were at first built after the manner of turntable, but have recently been constructed similar to other switches by Mr. Klose, of Rorshach. The circumstance of the cog-rail being elevated above the other rails, simplified the construction of this switch materially. The common rails are switched as ordinarily, and the cog-rail is gradually widened, the rounds being bent at the same time, so as to remain normal to each side bar at their ends until the rail has attained double its usual width, when it is continued as two separate cog-rails. The engines were at first built with vertical boilers; next with boilers that were level on an average grade (agreeing in these two steps with those followed on the Mount Washington Railway), but are now built with horizontal boilers, same as ordinary locomotives. Various methods have been devised for enabling the locomotives to work by adhesion of their smooth wheels, as well as by means of their cog-wheel drivers, and by means of either at will. No one of these has been permanently successful, however, so that the proper construction of a double engine of this sort is still a matter of experimental inquiry.

ENGINEERING STRUCTURES.

DESCRIPTION OF A PAPER DOME FOR AN ASTRONOMICAL OBSERVATORY. BY PROFESSOR DASCOM GREENE, TROY, New York. —An astronomical observatory has recently been erected for the Rensselaer Potytechnic Institute, through the liberality of Mr. E. Proudfit of this city. In maturing the plans, and supervising the erection of the building, I have introduced an improved method of constructing revolving domes, a brief account of which may not be without interest.

While making the preliminary inquiries, I ascertained that a dome of the dimensions required, constructed in any of the methods in common use, would weigh from five to tons, and require the aid of cumbersome machinery to revolve it. It therefore occurred to me to obviate this objection by making the framework of wood, of the greatest lightness consistent with the requisite strength, and covering it with paper of a quality similar to that used in the manufacture of paper boats; the principal advantages in the use of these materials being that they admit of great perfection of form and finish, and give extreme lightness, strength, and stiffness in the structure—prime qualities in a movable dome. A contract was accordingly made with Messrs. E. Waters & Sons, of this city, the well-known builders of paper boats, for the construction of the dome, and they have carried out the undertaking with great skill and success.

The dome is a hemisphere with an outside diameter of twenty-nine feet. The framework consists primarily of a circular sill which

forms the base, and two semi-circular arch girders set parallel to each other, four feet apart in the clear, and spanning the entire dome. These are firmly attached to the sill, and kept in a vertical position by means of knee-braces. The sill and girders are of seasoned pine, the former being 8¼ inches wide by 3½ thick, and the latter each 4½ by 3 inches.

The paper covering of the dome is made in sixteen equal sections, such that when set up side by side, their bases on the sill, and their extremities meeting at the top, they form a complete hemispherical surface. The frame-work of each section consists of three vertical ribs of pine, each 3¾ inches in width and ¾ of an inch thick, one at each side, and one mid-way between, and meeting at the apex. The paper was stretched over this frame-work as follows :

A wooden model of full size being made of that portion of the dome included within one of the sections, with a surface truly spherical, the frame-work of a section was placed in its proper position on the model, so that its outer edges formed part of the same spherical surface, and covered with shellac where it was to be in contact with the paper. The sheet of paper cut in the proper form was then laid on the model while moist, the edges turned down over the side ribs, and the whole placed in a hot chamber and left until thoroughly dry. In this way the several sections were dried off in succession over the same model. The paper used is of a very superior quality, manufactured expressly for the purpose by Messrs. Crane Brothers, of Westfield, Mass. Its thickness after drying is about one-sixth of an inch, and it has a structure as compact as that of the hardest wood, which it greatly excels in strength, toughness and freedom from any liability to fracture.

After being thoroughly painted, the several sections were ready to be set up side by side on the sill, and connected together by bolting through the adjacent ribs. The space between the arch girders being left uncovered on one side from the sill to a distance of two feet beyond the zenith, the upper ends of the sections required to be cut off and accurately fitted to the girders. The joints between sections were made weather proof by inserting a double thickness of heavy cotton cloth saturated with white lead paint. The adjacent side ribs were then bolted firmly together through the paper and cloth, the lower ends attached to the sill by angle irons, the upper ends bolted to the girders, and the lower edge of the paper turned under the sill and securely nailed. The joints were afterwards painted over on the outside. As the entire surface exposed is free from nail holes or other abrasions in the paper the structure promises, with an occasional coat of paint, to last for many years, and to form an effective and serviceable roof.

The four feet opening between the arch girders is covered by a shutter which is also of paper stretched over a wooden frame. With the exception of about two feet at the lower extremity this shutter is in a single piece. Attached to its sides are a series of iron rollers which run on a railway track of band iron laid down on the girders, by which means the shutter can be moved over to the opposite side of the dome. The wooden sides of the shutter have iron flanges attached to their lower edges, which project under the railway tracks, making the whole weather proof. The shutter is opened and closed by means of a windlass and wire rope.

The weight of the dome and its appurtenances is about 4,000 pounds. It is supported on six 8-inch balls which roll between grooved iron tracks, and can be easily revolved by a moderate pressure applied directly, without the aid of machinery.

ORDNANCE AND NAVAL.

REPEATING-RIFLES FOR THE FRENCH NAVY. —THE French authorities have recently made a careful trial of repeating arms with a view to adoption should one be found which, while serviceable in other respects, fulfilled the following conditions, which were put forth in March, 1877 :—(1) To fire the regulation metallic cartridge of the army. (2) To have the same trajectory and the same accuracy as the rifle model, 1874. (3) So constructed as to be used as an ordinary single shot arm, or, in other words, to admit of passing quickly and simply from single shot loading and firing to repeating and *vice versa*. (4) To be strong, not requiring too tender care, not to be exposed, from a breaking down of the repeating mechanism, to unserviceableness as a single shooter, to be dismounted, cleaned, and remounted without difficulty. On March 28th, 1877, the minister approved of this programme, and on September 14th he sent orders to Cherbourg to experiment with three types of repeaters, with detailed instructions as to the trials. These three arms were :—(1) The Hotchkiss ; (2) the Kropatschek ; (3) the Krag. To these three the board confined themselves.

The result of these trials showed that the magazine of the Hotchkiss was most quickly charged. The Hotchkiss also fires most rapidly; both in repeating and single shot fire the Kropatschek was not far behind it. The Krag does not seem to have been well understood and manipulated by the men.

The Kropatschek—modified—with eight cartridges in its magazine beat the Hotchkiss which had only six, while the Krag with nine cartridges was best of all. The time necessary to discharge this latter arm's magazine of nine rounds was 24.85 seconds, in which time the Kropatschek had on an average fired 8.9 cartridges per arm, and the Hotchkiss 7.9 starting with the magazine closed : with the magazine open 25 seconds were occupied, in which time the Krag fired 9, the Kropatschek 9.3 and the Hotchkiss 8.25 rounds on an average. Single shot fire proved better than recharging the magazine and repeating continually. The minimum times taken to fire off the magazines, at the conclusion of the experiments, when the men were expert, were as follows :—Hotchkiss—6 rounds—in 10 seconds ; Kropatschek, modified—8 rounds—in 14 seconds ; Krag—9 rounds—in 17 seconds ; giving an average time

per round of 1.66, 1.75, and 1.88 seconds respectively.

Finally, it was concluded that the Hotchkiss rifle is the easiest and quickest in charging the magazines; then the Kropatschek; and last, Krag. As to rapidity of fire, the Hotchkiss and Kropatschek are about equal. Large magazines have a great advantage; the magazine once empty. it is best not to attempt to refill it till leisure gives the opportunity.—*Engineer.*

BOOK NOTICES.

SPECIMEN BOOK OF ONE-HUNDRED ARCHITECTURAL DESIGNS, SHOWING PLANS, ELEVATIONS, VIEWS, AND INCLUDING BILLS OF MATERIALS. New York: A. J. Bicknell & Co. For sale by D. Van Nostrand. Price, $1.00.

The illustrations of this book are selections from various works on Architecture. The views are well printed on good paper, and constitute the book. There is no descriptive text worth mentioning.

THE STUDY OF ROCKS. By FRANK RUTLEY, F.G.S. New York: D. Appleton & Co. For sale by D. Van Nostrand. Price $1.75.

This is a late addition to the series of "Text-Books of Science."

It is adapted for beginners in Petrology and only supposes the merest rudimentary knowledge of Geology, and a fair amount of mechanical skill to be applied to the preparation of specimens for the microscope.

The method of the author is logical and clear, such that the ordinary student can follow with profit without requiring the aid of an instructor, provided only that he can detect the common minerals by the ordinary processes, and can also work the microscope.

Some diagram "schemes" of deviations from well known rocks are excellent. Other illustrations of the book are good and fairly numerous.

EXPERIMENTS ON REPEATING-RIFLES BY A BOARD OF FRENCH NAVAL OFFICERS. Translated by Lieut. Theo. B. M. Mason. New York: Office of *Army and Navy Journal.*

The question of superiority of the repeating over the single short rifle has been the subject, in Europe especially, of long discussions.

The French Navy decided to arm the sailors with a magazine gun, and began for that purpose a systematic course of experiments, of which the report is a full account.

The Report first presents the reasons for using repeating rifles, quoting from eminent military authorities. The description of the arms experimented with next follows. These rifles were the Hotchkiss (American) Krag (Norwegian) and Kropatshick (Austrian). The reasons for finally adopting the latter are fully set forth.

Many readers who are not in any degree inclined to military tastes will read this report with interest.

FILTRATION OF POTABLE WATER. By WM. RIPLEY NICHOLS. New York: For sale by D. Van Nostrand. Price $1.50.

The filtration of water on a large scale, has been but little practiced in this country. The necessity of such a treatment in the case of many sources of supply is very generally recognized, but the practice is but little more than a rude kind of straining.

Professor Nichols has clearly specified the circumstances which render filtering advisable, and has described with exceptional fullness the methods by which it may be accomplished.

The illustrations of working filters, both for town and for family supply, are given with an attention to detail which makes them working plans.

The work is indispensable to engineers who manage or project water supply systems, and will prove equally valuable to those who regulate such supply with reference to sanitary precaution.

LES PONTS DE L'AMERIQUE DU NORD. Par L. ANT. COMOLLI. Ingenieur. Paris: Ambroise Lefevre. For sale by D. Van Nostrand. Price, $18.00.

This is a quarto volume of 450 pages, accompanied by an atlas of 54 plates.

The text is divided into two nearly equal parts. The first gives the theory of bridge building as presented by American writers, a large portion of Shreve's excellent work occupying a prominent place. Whipple and Merrill are also freely quoted. The second part presents examples of American Bridges, and in this part also well known American books including descriptive papers from several sources.

Hildebrand's work on "Cable Making for Suspension Bridges," published originally in this Magazine, is given entire.

The Atlas of Plates is an excellent collection of plans of American bridges with well drawn details. To engineers who would like native engineering literature done into French the opportunity is a good one.

MISCELLANEOUS.

Two great but frequently-recurring canal questions are again said to be occupying much attention in America. It is once more said that the route for the Panama Canal has been fixed. Lake Nicaragua is to be utilized, and a cut is to be made through the mountain chain between Nicaragua and Brito into the lake. From the northeastern part of the lake exit will probably be through a cut parallel to the river San Juan into the Gorgon Bay. The other canal is the long-talked-of ship canal to connect Chesapeake Bay with Delaware Bay, and shorten the water route from Baltimore to New York and Europe some 225 miles. The canal will be seventeen miles long, 100 feet wide and 25 feet deep, and the estimated cost is stated as £800,000. The promoters state that the present commerce of Baltimore would give to the canal an income of $160,000 from the authorized rate of toll—10d. per ton. The canal is to follow the valley of the Sassafrs, and be without locks.